

UNCLASSIFIED

AD NUMBER

AD468061

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies and their contractors;
Administrative/Operational Use; JUL 1965. Other requests shall be referred to National Aeronautics and Space Administration, Washington, DC.

AUTHORITY

RSIC ltr 20 Dec 1965

THIS PAGE IS UNCLASSIFIED



Avco EVERETT

RESEARCH LABORATORY

a division of
AVCO CORPORATION

**AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED
FLUXES INCLUDING LINE CONTRIBUTIONS
AND SELF ABSORPTION**

R. A. Allen

RESEARCH NOTE 561

July 1965

supported jointly by

HEADQUARTERS

**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
OFFICE OF ADVANCED RESEARCH AND TECHNOLOGY**

Washington 25, D. C.

under Contract No. NASw-748

ADVANCED RESEARCH PROJECTS AGENCY

monitored by the

ARMY MISSILE COMMAND

UNITED STATES ARMY

Redstone Arsenal, Alabama

under Contract No. DA-01-021-AMC-12005 (Z)

(part of Project DEFENDER)



SECURITY

MARKING

The classified or limited status of this report applies to each page, unless otherwise marked.

Separate page printouts MUST be marked accordingly.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

DDC AVAILABILITY NOTICE

**QUALIFIED REQUESTORS MAY OBTAIN COPIES
OF THIS REPORT FROM DDC**

**This research is a part of Project DEFENDER,
sponsored by the Advanced Research Projects
Agency, Department of Defense, under ARPA
Order No. 525.**

RESEARCH NOTE 561

AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES
INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION

by

R. A. Allen

AVCO-EVERETT RESEARCH LABORATORY
a division of
AVCO CORPORATION
Everett, Massachusetts

July 1965

supported jointly by

HEADQUARTERS
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
OFFICE OF ADVANCED RESEARCH AND TECHNOLOGY
Washington 25, D.C.

under Contract No. NASw-748

ADVANCED RESEARCH PROJECTS AGENCY
monitored by the
ARMY MISSILE COMMAND
UNITED STATES ARMY
Redstone Arsenal, Alabama

under Contract No. DA-01-021-AMC-12005 (Z)
(Part of Project DEFENDER)

ABSTRACT

Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000°K to 23,000°K and over a density range from 10^{-6} to 10 atmospheres. The calculations are presented in graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic line and free-bound transitions are important contributors at elevated temperatures and also generally undergo significant amounts of self-absorption. Consequently calculations for these two classes of radiation have been made with slab thickness as an added parameter. The lines, free-bound and free-free radiation have also been split into the two spectral regions for the radiation flux above and below 2000 Å.

INTRODUCTION

These tables are designed primarily to aid in the estimation of radiation heating fluxes to be expected from the gas cap region generated ahead of hypersonic re-entry vehicles entering the earth's atmosphere. The calculations have been carried out for infinite air slabs of finite thickness since this geometry lends itself most readily to practical applications. For instance, radiation fluxes at the stagnation point of a vehicle estimated on the basis of an infinite slab approximation do not exceed the value based on better geometric considerations by not more than about 15%.¹ This over-estimation is also quite compatible with the general accuracy to which oscillator strengths are known.

MOLECULAR RADIATION

A. Diatomic Molecules

For the purposes of making these radiation tables,² we have made use of the expression developed by Keck³ et al and Patch⁴ for the radiation intensity emitted from an optically thin gas sample at the thermodynamic equilibrium. This expression is, after replacing the constants by their proper values:

$$\frac{dI}{dV d\Omega d\lambda} = 3.19 \times 10^{-34} \left| \frac{R(\bar{r})}{ea_0} \right|^2 N \phi \tilde{\nu}^6 \frac{hc}{kT_r} e^{-\frac{hc \tilde{\nu}_{00}}{kT_e}} \frac{w}{\text{cm}^3 - \mu - \text{ster}} \quad (1)$$

The symbols are as those given in Ref. 5 where $R(\bar{r})$ is the dipole matrix element for the transition and may be a function of the internuclear separation. a_0 is the radius of the first Bohr orbit and e is the charge of the electron. N is the number of molecules in the absorbing state which is not necessarily the ground state, and h , c , and k are the usual constants. T_r , T_v and T_e are the rotational, vibrational and electronic temperatures, respectively. The absorption oscillator strength is related to the matrix element by the expression,

$$f_{v''v'} = \left(\frac{\tilde{\nu}}{3R_\infty} \right) \left| \frac{R(\bar{r})}{ea_0} \right|^2 \quad (2)$$

where R_∞ is the Rydberg.

As used in formulas (1) and (2), the electronic transition moment, $\left| \frac{R(\bar{r})}{ea_0} \right|^2$ is related to the radiative lifetime for spontaneous emission, τ , by the following expression

$$\frac{1}{\tau} = \frac{8\pi^2 r_0 c \tilde{\nu}_e^3 g''}{3 R_\infty g'} \left| \frac{R(\bar{r})}{ea_0} \right|^2 \quad (3)$$

where g' and g'' are the electronic degeneracies (either 1 or 2 for diatomic molecules) of the upper and lower states respectively and

$$\bar{\nu}_e^3 = \sum_{v''} \bar{\nu}^3 q_{v',v''} \quad (4)$$

Note should be made that the electronic transition moment has been defined differently by various authors,^{5,6,7,8} the differences occurring in the way that the electronic degeneracies appear in the expression (3) that relates the lifetime for spontaneous emission to the electronic transition moment.

ϕ is called the spectral distribution function and gives the spectral details of the band system. It is calculated from a knowledge of the Frank Condon factors and the spectroscopic constants.

$$\phi = \frac{kT_r}{hc} \frac{1}{Q_v'' Q_r'' (B_e' - B_e'')} \sum_{\epsilon_r' \geq 0} q_{v',v''} e^{-\left(\frac{\epsilon_v' k}{T_v} + \frac{\epsilon_r' k}{T_r}\right)} \quad (5)$$

where: $Q_v'' = [1 - \exp(-hc\omega_e''/kT_v)]^{-1}$ is the vibrational partition function for the absorbing state, (the ' refers to the excited electronic or emitting state, and the '' refers to lower electronic or absorbing state);

$Q_r'' = kT_r/hcB_e''$ is the rotational partition function for the absorbing state;

$q_{v',v''}$ is the Condon factor for the transition from v' to v'' ;

$\tilde{\nu}_v = \omega_e(v + 1/2) - \omega_e x_e(v + 1/2)^2 + \omega_e y_e(v + 1/2)^3$ is the wave number of the v^{th} vibrational level;

$\tilde{\nu}_{v',v''} = \tilde{\nu}_{00} + (\tilde{\nu}_{v'} - \tilde{\nu}_{0'}) - (\tilde{\nu}_{v''} - \tilde{\nu}_{0''})$ is the wave number for the vibrational transition between states v' and v'' ;

$\epsilon_{v'} = hc(\tilde{\nu}_{v'} - \tilde{\nu}_{0'})$ is the vibrational energy relative to the ground vibrational state for the emitting state;

$\epsilon_{r'} = B_{e'} / (B_{e'} - B_{e''}) hc(\tilde{\nu} - \tilde{\nu}_{v',v''})$ is the rotational energy for the emitting state, with the approximation made that the rotational quantum number J does not change in a given transition.

In a transition, J can change by $+1, 0$ or -1 giving R, Q and P branches, respectively. However, for increasing rotational temperatures where high J values are more heavily weighted or where the spectrum is observed with coarse resolution, the effect of $\Delta J \pm 1$ is washed out. The degree to which this approximation matches the detailed calculations for the rotational structure is discussed in reference 9. Although it lacks detail within an interval $\Delta \tilde{\nu} \approx B_e^2 / \Delta B_e$ of the band head $\phi \tilde{\nu}^6$ gives a good description of the vibrational structure.

ϕ and related parameters were calculated for the various radiators which should contribute in air for $T_r = T_v = T_e$, and appear in references 2 and 9. These radiators along with the recommended oscillator strengths are listed in Table 1.

For the sake of simplicity and also because of the general lack of information to the contrary, no internuclear separation dependence for the electronic transition moments has been assumed in calculating the total radiation to be expected from the molecular band systems listed in Table I. The integration over wavelength was carried out by a computer program which calculated $\phi \lambda^{-6}$ and integrated over wavelength. The computer output for these calculations are given in Part I² of these radiation tables and also in reference 9. The reference sources for the oscillator strengths are also listed and are felt to be the last values to use although several of them are still open to question, notably those for the $N_2(1+)$ and N_2 Birge Hopfield systems.

An electronic transition moment of 1.0 was used for the $N_2(1+)$ system since Wray²⁸ has found this value to use in order to best phenomenologically fit air data. An electronic transition moment of 1.4 for the $N_2 b^1\Sigma \rightarrow X$ Birge Hopfield is used with no value assigned to the $b^1\Pi \rightarrow X$ transition; however we found that the best fit to the temperature dependence of the data of Wray³⁵ is given by assuming that the main N_2 component in the ultraviolet is the $b^1\Sigma \rightarrow X$ transition. For discussions of the oscillator strengths pertaining to the other radiators attention should be given to the appropriate references in Table I.

The calculations for the band system in Figures 1 - 8 are carried out for the flux from one side of an infinite slab (i.e. 2π ster) of 1 cm thickness

TABLE I

Molecular Band Radiation

	$\left \frac{R(\bar{r})}{ea} \right ^2$	τ sec	f_{00}	Ref.	Key
N ₂ (1+)	.1 (1.0 Air)	1.1×10^{-5}	.003	5, 28	3
N ₂ (2+)	.7	2.7×10^{-8}	.06	5	1
N ₂ Birge Hopfield #1 b ¹ Π → X				32	
N ₂ Birge Hopfield #2 b ¹ Σ → X	1.4	$.71 \times 10^{-9}$.43	17, 32	2
N ₂ Lyman Birge Hopfield	1.77×10^{-5}	1.7×10^{-4}	$.37 \times 10^{-5}$	7	9
N ₂ ⁺ (1-)	.45	$.67 \times 10^{-7}$.035	5, 9	6
N ₂ ⁺ (Meinel)	.45	3×10^{-6}	.012	7, 34	7
NO β	.08	$.67 \times 10^{-6}$.01	5	4
NO γ	.025	1.16×10^{-7}	.003	5	5
O ₂ Schumann-Runge	0.5	$.82 \times 10^{-8}$.075	5	8
NO Infrared Bands				33	13
NO ₂ Thermal				29	15
NO ₂ Recombination				29	14

Electron Recombination and Scattering Radiation

N ⁺ + e → N + hν	Hydrogen like model with Bethe & Salpeter continuum $\bar{\gamma}$ numbers and photoionization cross section of 10^{-17} cm ² for 2 p states	17, 18	12
O ⁺ + e → O + hν			
N ⁺ + O ⁺ free free	Unsold expression with effective Z ² of 1.5	17	12
N ⁺⁺ + O ⁺⁺ fb + ff	Unsold expression with effective Z ² of 4		18
O + e O ⁻ + hν	Photoionization cross sections	30	10
N ₂ ff	Unsold expression with Z ² = .022	31	16
N _{ff}	Unsold expression with Z ² = .009	31	17

Atomic Line Radiation

N, O		26	11
------	--	----	----

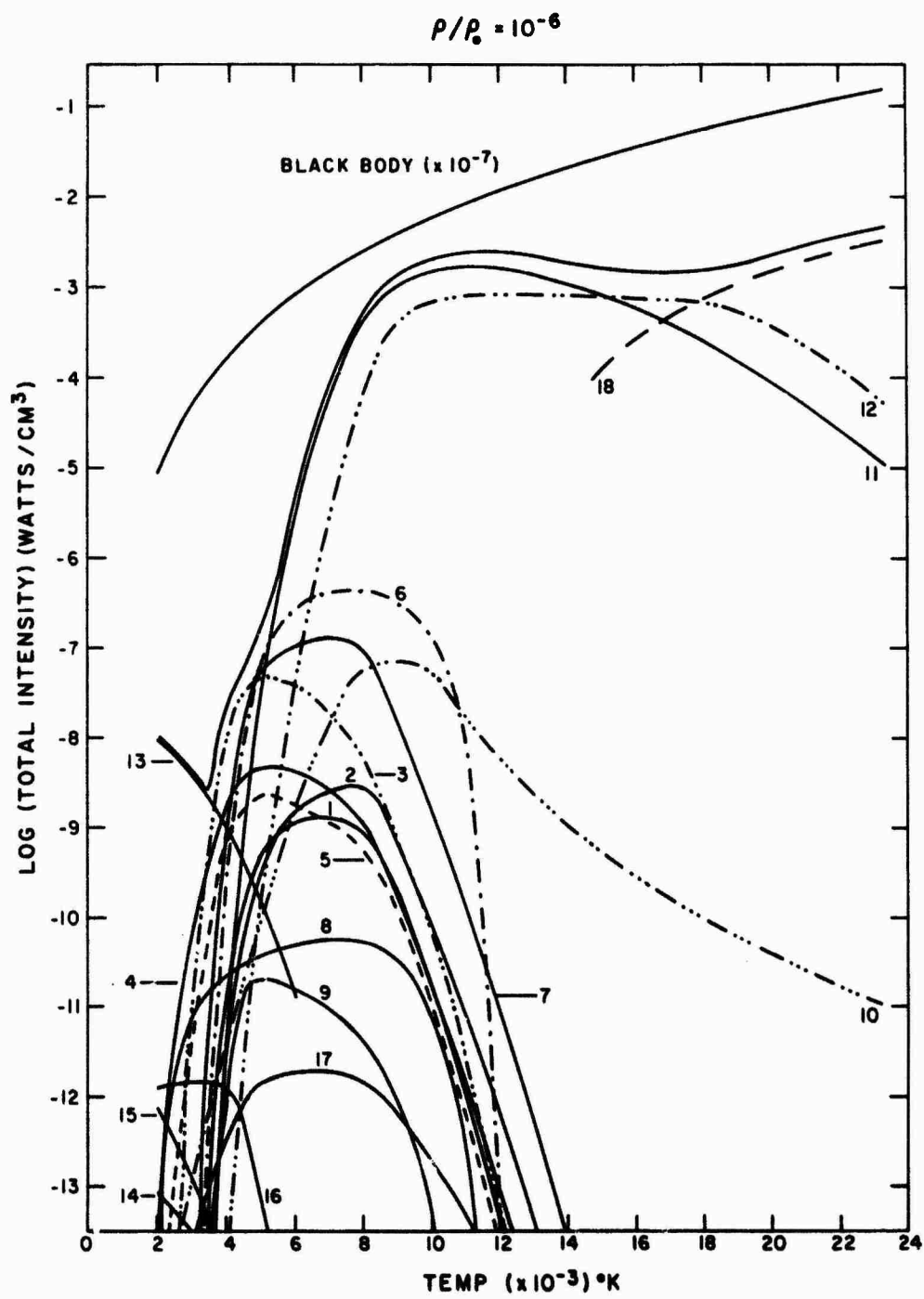


Fig. 1 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^{-6} atmospheric.

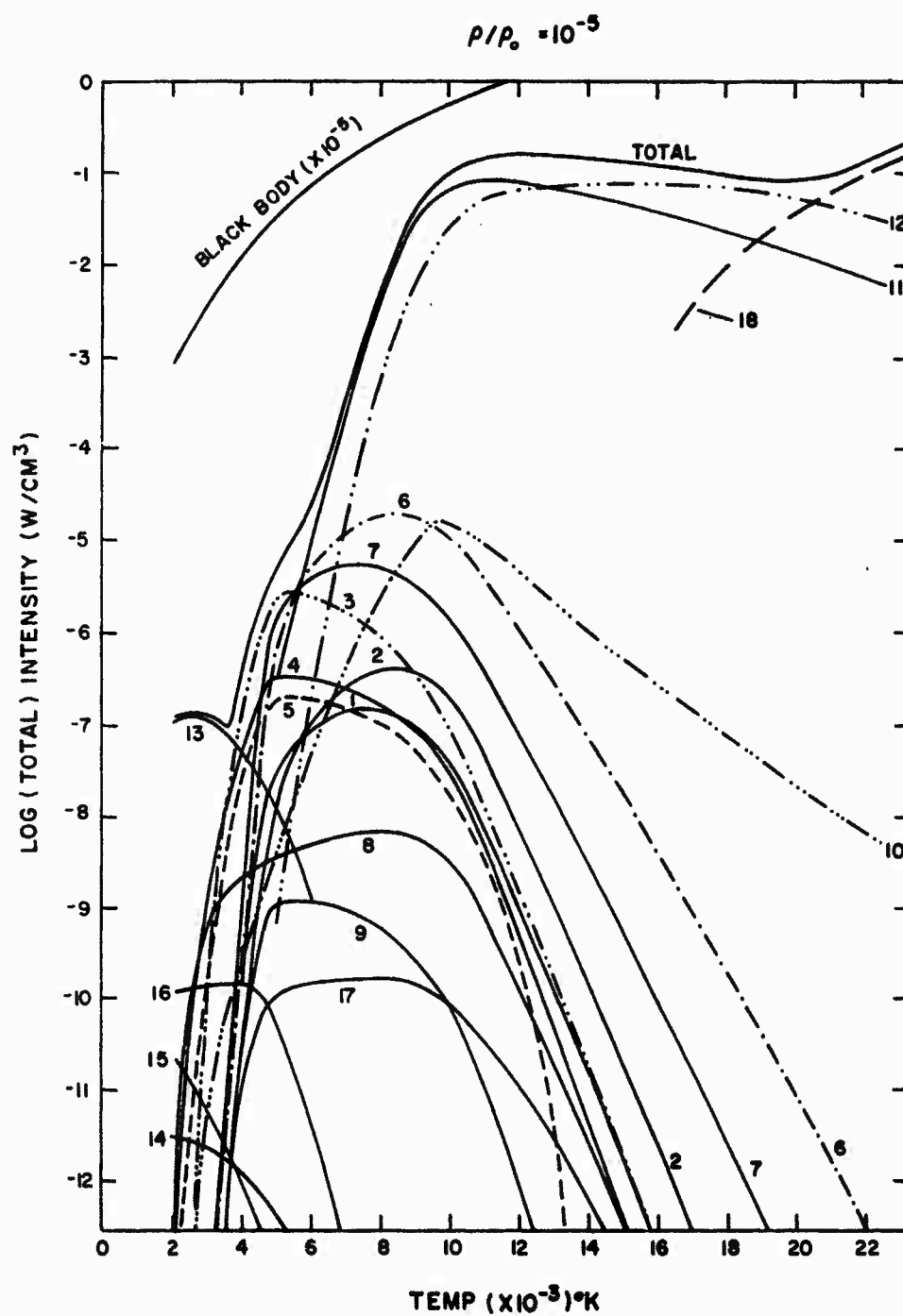


Fig. 2 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^{-5} atmospheric.

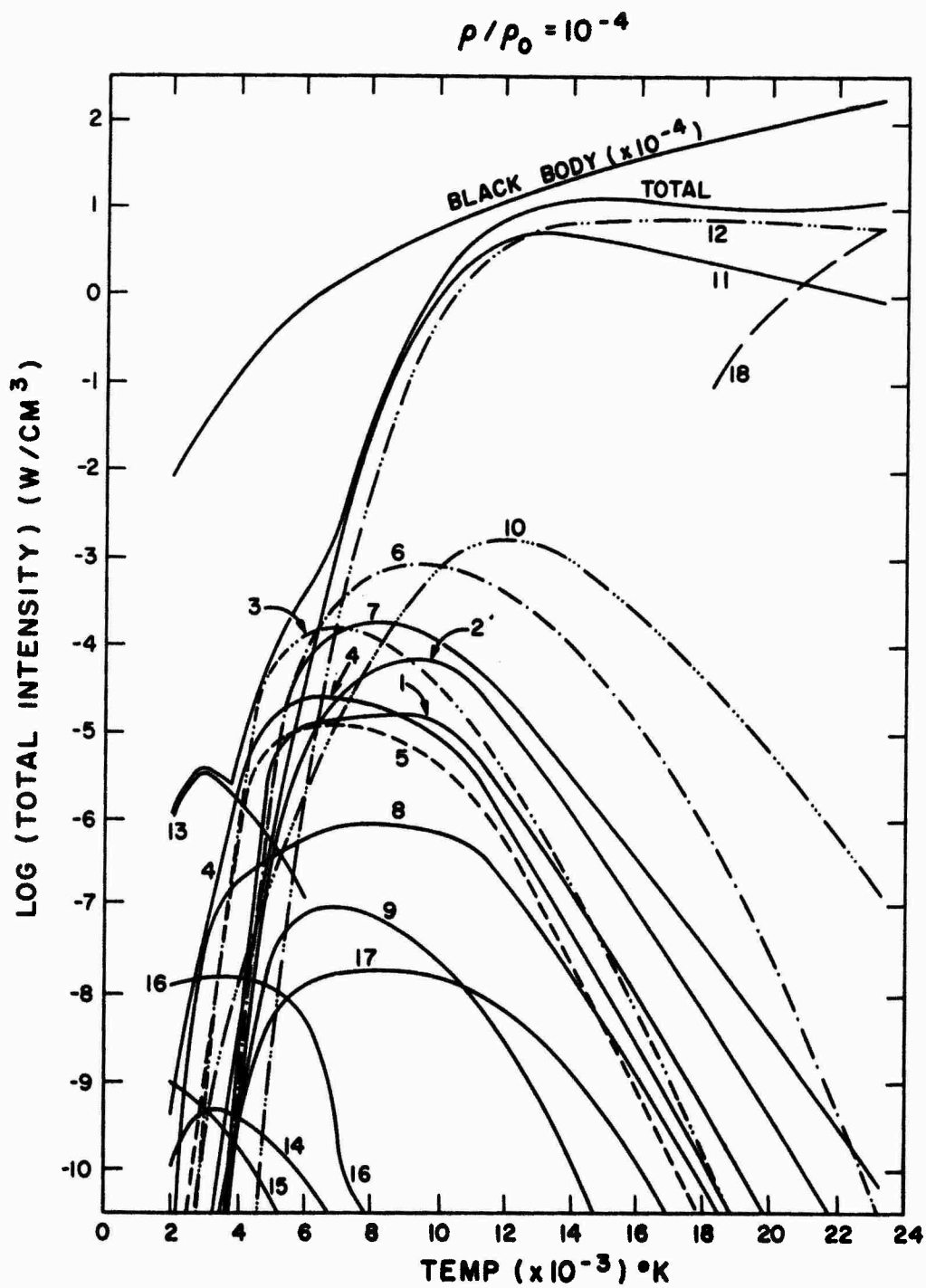


Fig. 3 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^{-4} atmospheric.

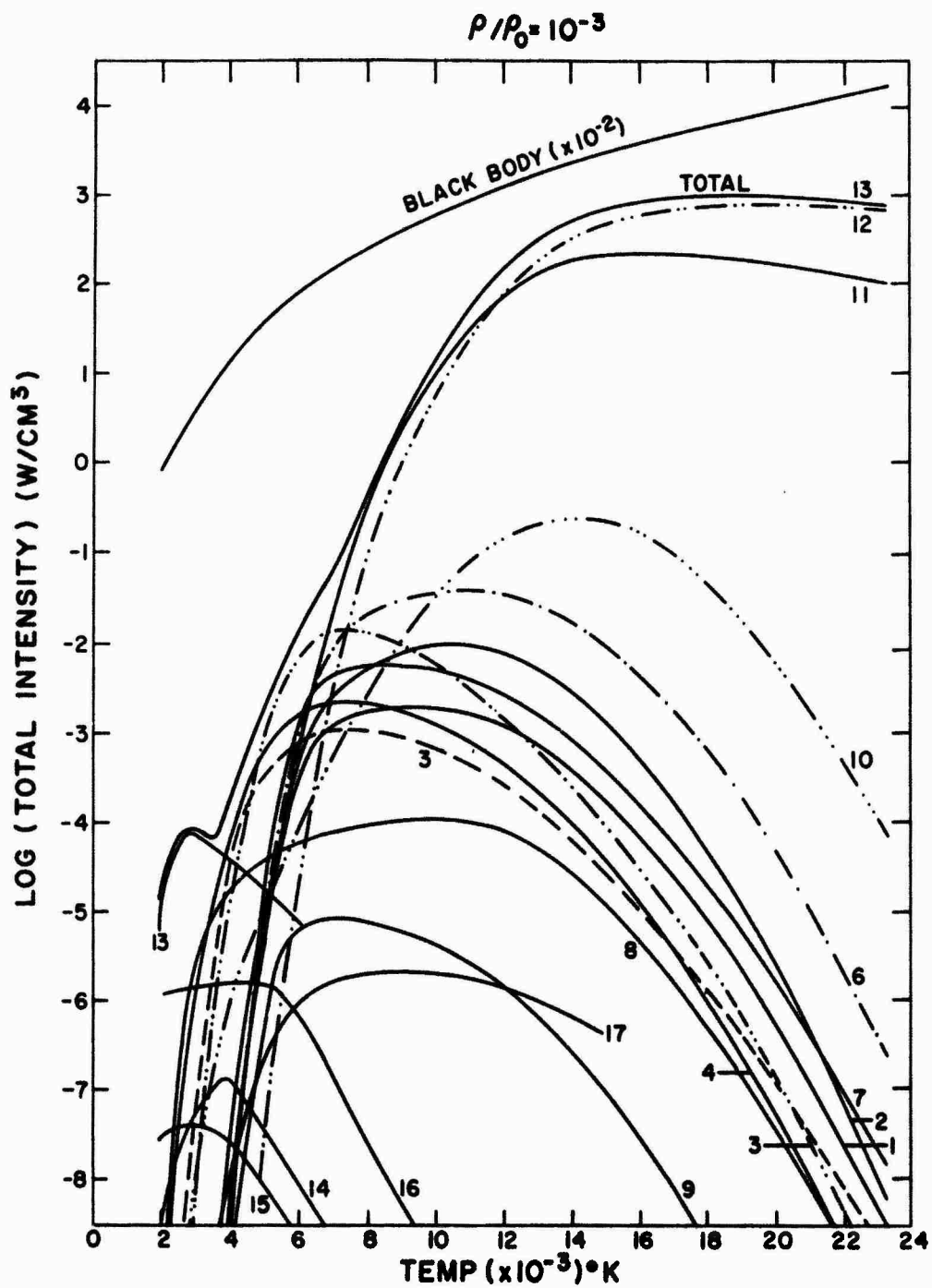


Fig. 4 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^{-3} atmospheric.

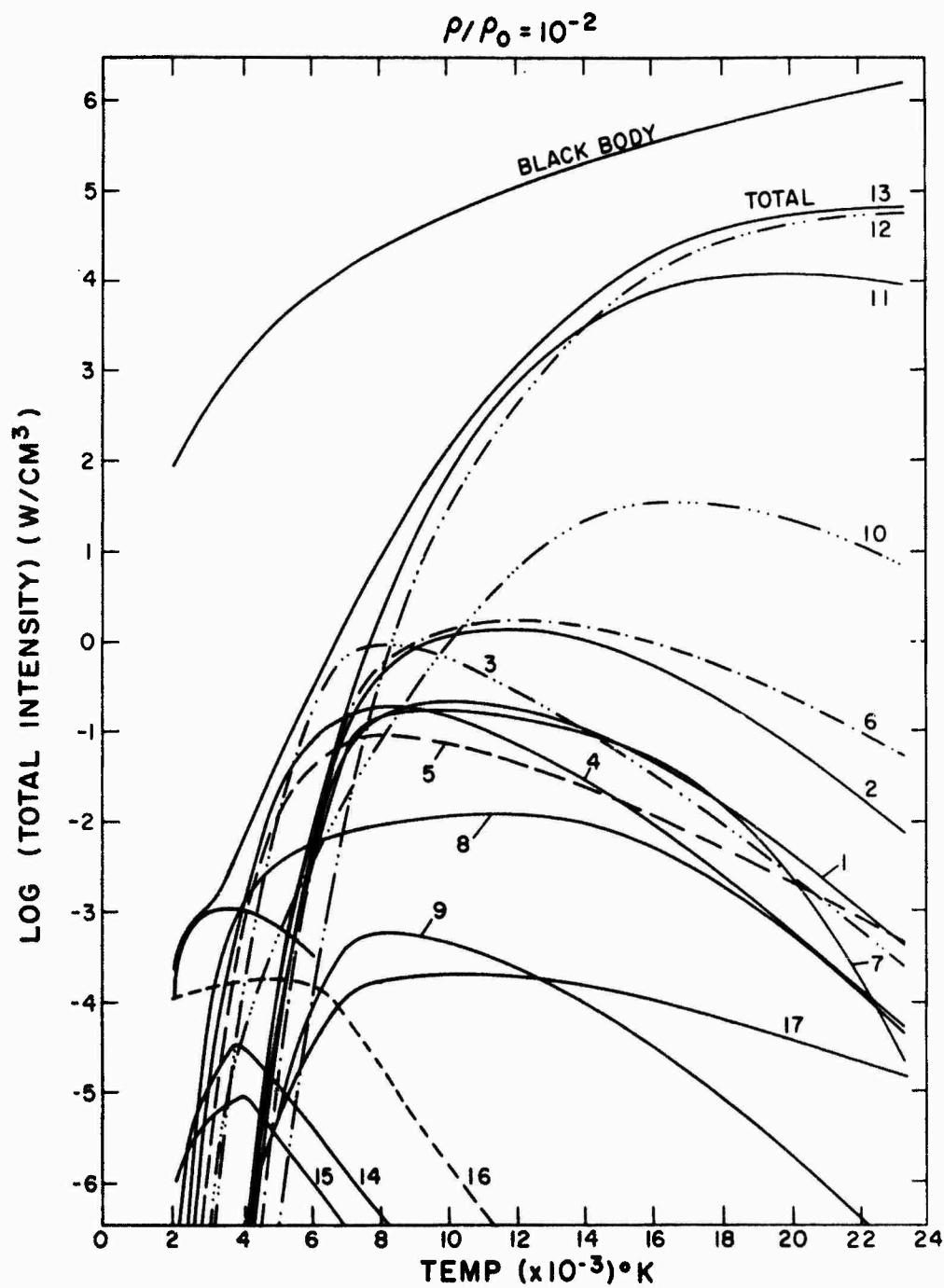


Fig. 5 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^{-2} atmospheric.

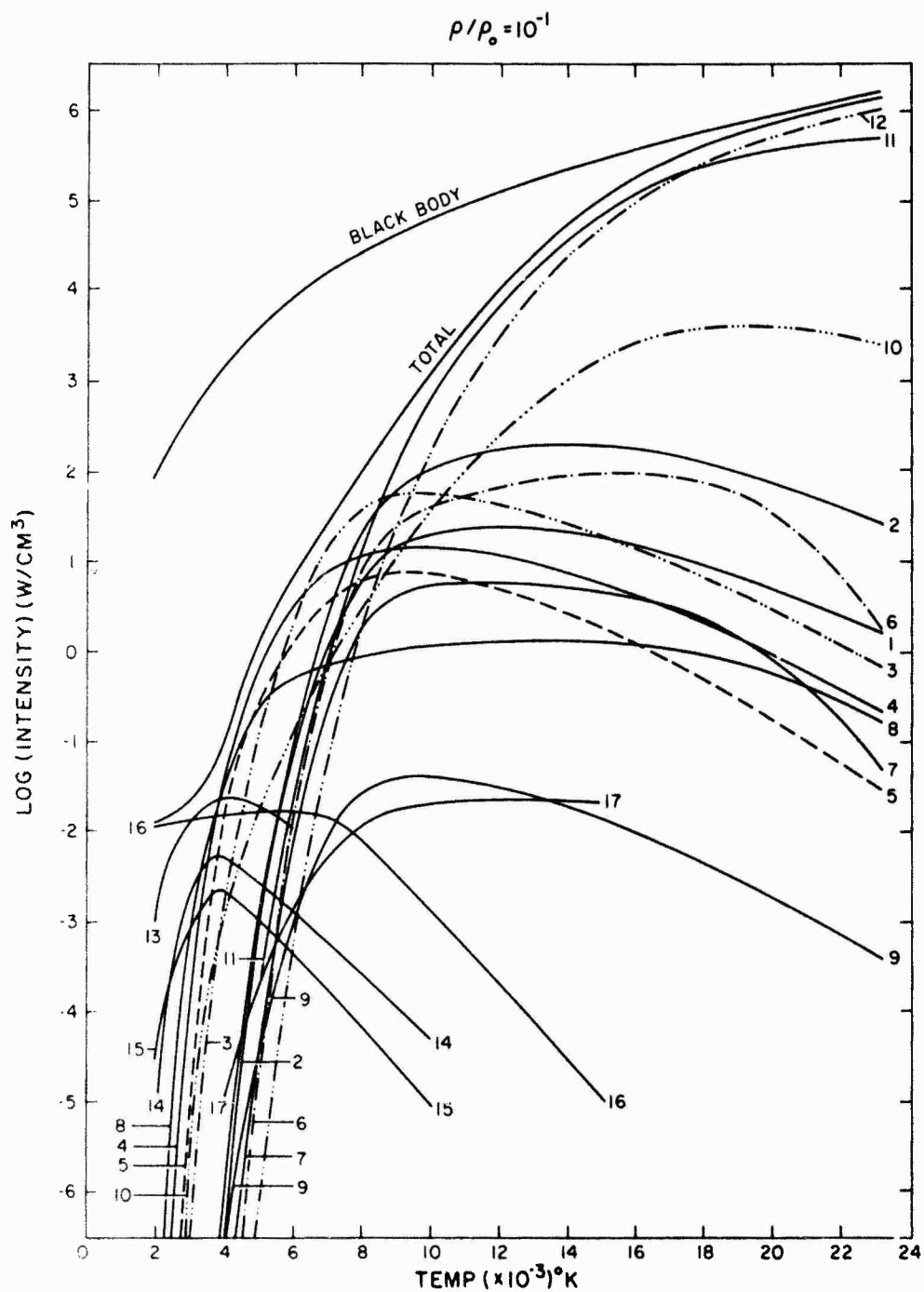


Fig. 6 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^{-1} atmospheric.

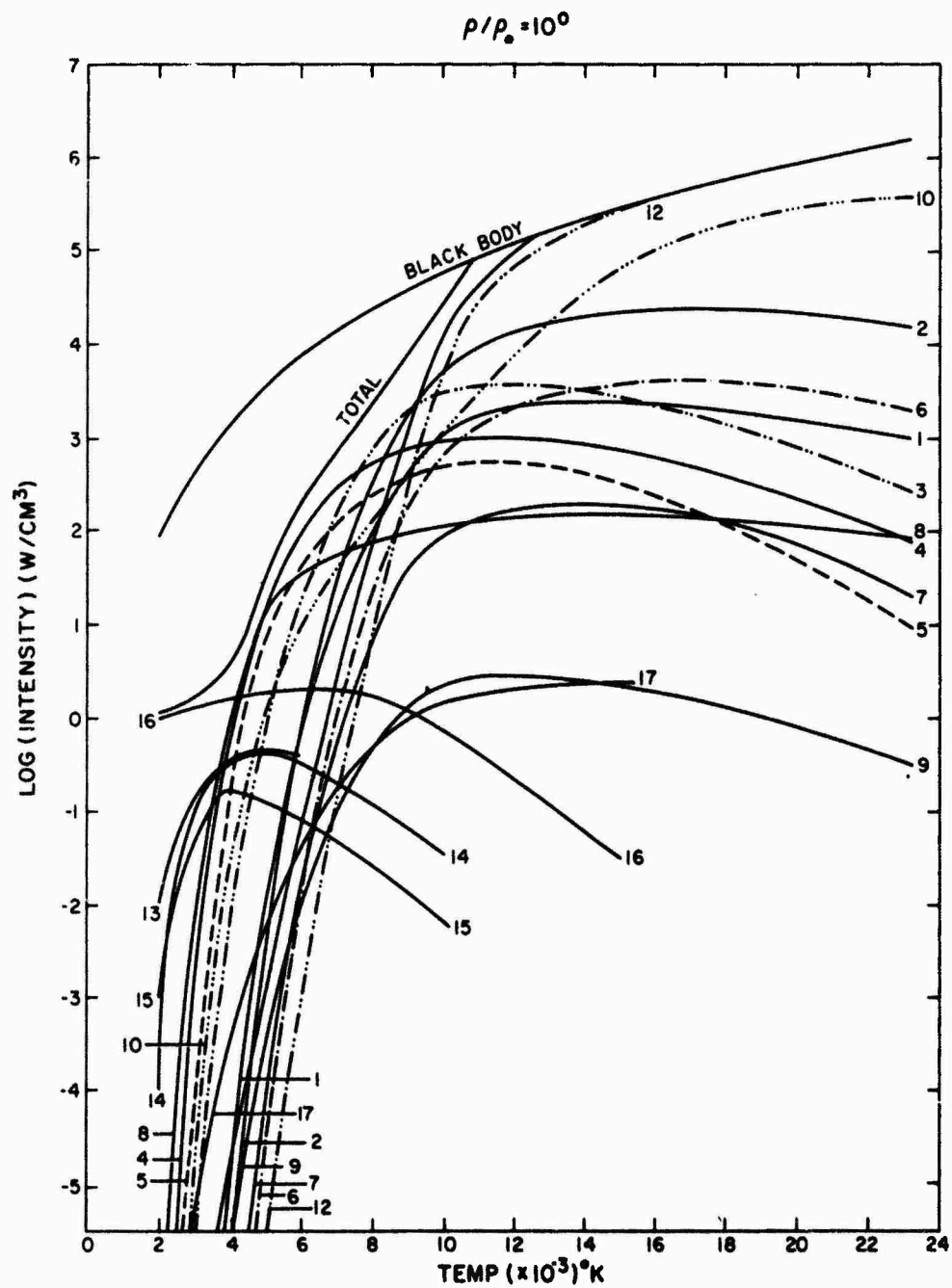


Fig. 7 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^0 atmospheric.

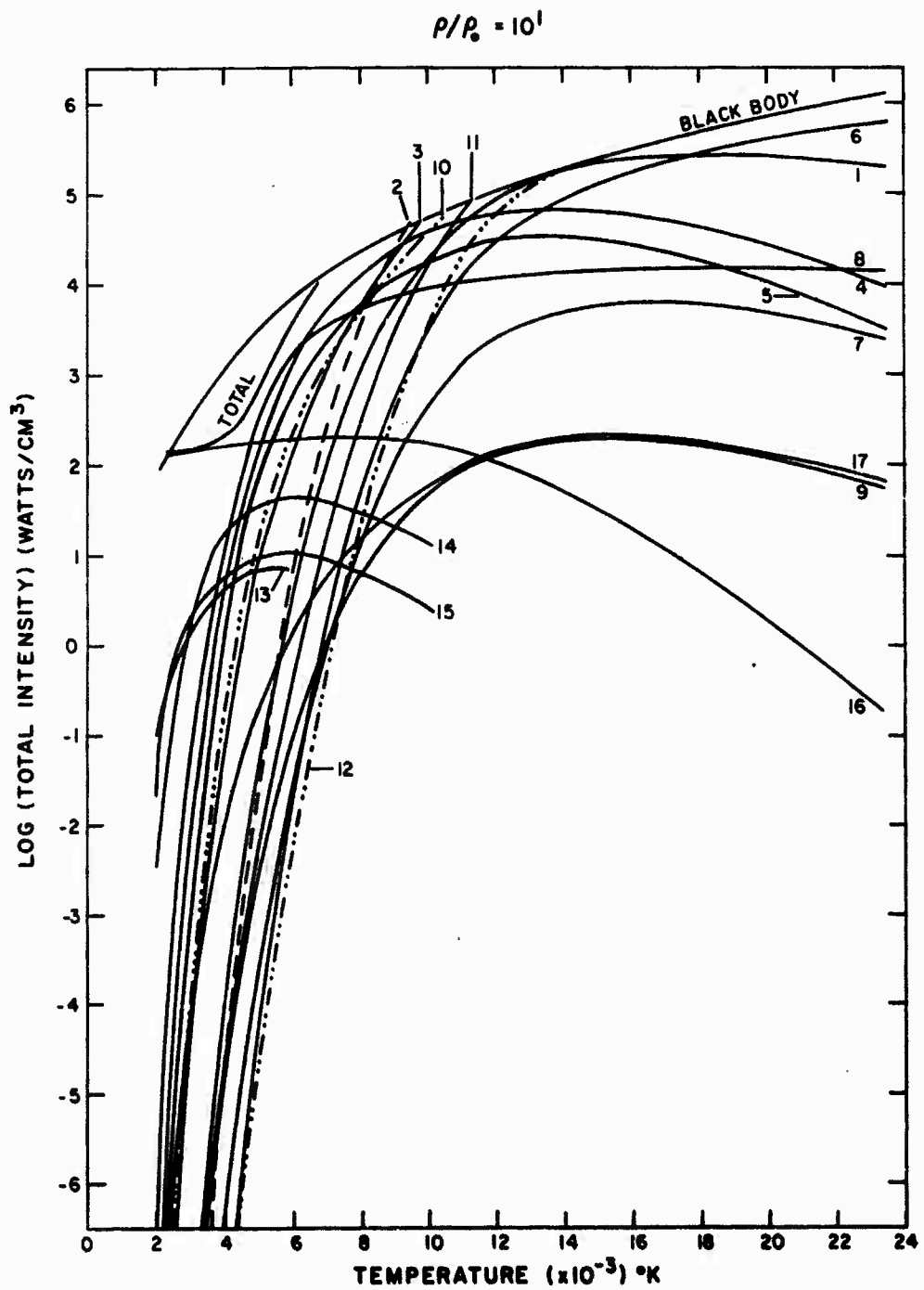


Fig. 6 Radiation calculations for the flux from one side of an infinite air slab 1 cm thick at a density of 10^1 atmospheric.

and also with the assumption that the band systems are optically thin until they are cut off by the black body. This simplification can lead to an over-estimation of radiation fluxes when the total gas emissivity calculation approaches unity and when the average line width to spacing ratio is small.

Fairbairn¹⁰ has performed a detailed line by line calculation over several sequences of the CN and C₂ violet systems with the line width and shape as a parameter, and is an example of the type of calculation which is needed to properly account for self-absorption of band systems when gas emissivities calculate near unity.

The species concentrations which were used in order to perform these radiation calculations are from the data published in reference 11 for temperatures up to 15,000°K and reference 12 for temperatures up to 24,000°K.

B. NO₂ Radiation

Radiation from NO₂ comes about by two processes. One that produced by an electron recombination with NO forming NO₂ and a photon and also NO₂ radiating from an electronic transition. We have chosen the formulas given by Leavitt¹³ to describe the radiation from these two sources.

FREE-BOUND AND FREE-FREE RADIATION

A. N^+ and O^+ Radiation

A calculation procedure was developed which used the continuum oscillator strengths of Bethe and Salpeter¹³ to make the free-bound radiation predictions for N^+ and O^+ for all transitions emitting at a wavelength longer than 2000 Å. This calculation proceeds on the basis that at thermodynamic equilibrium the emission can be related to the black body intensity by the photoabsorption coefficient, and that the excited electronic states of N and O can be approximated by a hydrogen like model.

The classical wave number dependence for a hydrogen photo-absorption edge is $\frac{1}{\tilde{\nu}^3}$. The f sum rule also states that $\int_0^\infty \tilde{\nu} d\tilde{\nu} = \pi r_0 f$ where r_0 is the classical electron radius and f is the oscillator strength. Therefore,

$$\sigma = \pi r_0^2 \frac{\tilde{\nu}_{nl}^2}{\tilde{\nu}^3} \bar{f}_{nl} \quad (6)$$

where $\tilde{\nu}_{n,l}$ is the wave number of the photoabsorption edge of atomic state n,l and \bar{f}_{nl} is the continuum oscillator strength of state of principal quantum number, n , and azimuthal quantum number, l . Energy levels for allowed transitions of the bound electron going to a free electron and the ion in its ground state were taken from the tables of Moore¹⁴ to locate the position of photoabsorption edges. The continuum oscillator strengths in conjunction with equation (6) were used to obtain the photoabsorption cross section for each level of the atom versus wave number. Contributions from all levels were then summed. The continuum oscillator strengths for all n states less than 5 were from Bethe and Salpeter, and for all other states the following expression was used with the constraint that \bar{f} must be equal to or less than 1.

$$\bar{f} = \frac{16 R^2}{3 \sqrt{3} \pi \tilde{\nu}_{n,l}^2 (2J+1)n^3} \quad (7)$$

Formula (7) can be obtained from the usual expression for the free-bound cross absorption cross section for hydrogen¹⁵ and equation (1). $\tilde{\nu}_{n,l}$ is the wave number of the photoabsorption edge for a given state, J and n are the inner and principle quantum numbers, respectively. R is the Rydberg expressed in wave numbers.

From the photoionization cross section, the radiation emitted from an equilibrium gas sample of thickness L perpendicular to the surface is given by

$$\frac{dI}{dA d\tilde{\nu} d\Omega} = BB_{\tilde{\nu}\perp} (1 - e^{-\sigma_{n,l} N_{n,l} L}) (1 - e^{-hc \tilde{\nu}/kT}) \quad (8)$$

If the gas is optically thin, the resultant expression for the radiation per unit volume per 2π ster becomes

$$\frac{dI}{dV d\tilde{\nu} d\Omega} = 2\pi N_{n,l} \sigma_{n,l} BB_{\tilde{\nu}\perp} (1 - e^{-hc \tilde{\nu}/kT}) \quad (9)$$

$BB_{\tilde{\nu}}$ is the black body intensity perpendicular to the surface and $(1 - e^{-hc \tilde{\nu}/kT})$ is the factor which properly accounts for induced emission. $\pi BB_{\tilde{\nu}\perp} = BB_{\tilde{\nu}}$ where $BB_{\tilde{\nu}}$ is the black body intensity radiated from a surface over 2π ster. h and c are the usual constants, $N_{n,l}$ is the concentration of absorbing atoms in state n,l and $\sigma_{n,l}$ is the photoabsorption cross section of state n,l . A computer program was written to calculate the free bound radiation from optically thin samples of N^+ and O^+ using the procedure outlined above.

A comparison between such a calculation and the Unsold¹⁶ expression for the sum of the free bound and free free ion radiation

$$\frac{dI}{dV d\tilde{\nu} d\Omega} = 1.63 \times 10^{-35} T^{-1/2} N_e N_i Z^2 \frac{W}{\text{cm}^2 \text{ster}} \quad (10)$$

with an effective Z^2 of 1 is shown in reference 17. Predictions for the visible continuum radiation from air using this calculation procedure are in reasonable agreement with measurements.¹⁷ In the vacuum ultraviolet however, free-bound transitions in air involve ground states or near ground states for the absorbing atom. These electronic states are least likely to be coulombic and are also states which have equivalent electrons. The hydrogen like oscillator strengths are therefore not applicable to these transitions for N and O although they are reasonable to use in predicting free bound transitions occurring at longer wavelengths. This fact seems to be bourn out by the shock tube data obtained in the visible.¹⁷ We have chosen to use a photoabsorption cross section for all allowed transitions from 2p states of N and O to be 10^{-17} cm^2 with no wavenumber dependence. Bates and Seaton¹⁸ calculate approximately this value for the ground atomic state of N and is also a value which has been inferred from measurements by Ehler & Weissler.¹⁹ The radiation flux calculations for most gas conditions and geometries was not very sensitive to what cross section was used for photoionization of O in the vacuum ultraviolet. All allowed free-bound transitions for O involving the recombination of an electron and O^+ in the 2p states occur at wavelengths below 910 \AA . This wavelength region is not only dominated by $N^+ + e$ processes but in situations where radiation heating is important is also usually strongly self absorbed. Consequently, the radiation is limited by the black body intensity.

Self absorption serves to reduce the free-bound radiation in the ultraviolet under many conditions over the spectral region covering the 2p photoabsorption edges of N. Use has been made of the expression of Olfe²⁰ which relates the flux from one side of an infinite gas slab to a continuous absorption coefficient. The results of calculations for the radiation emitted in two spectral regions are given in figures 9 - 32 for the region below and above 2000 \AA respectively. The growth curve of total intensity with slab thickness does not obey a simple growth law since part of the spectrum is optically thin while part is not, consequently the results are given with slab thickness as a parameter.

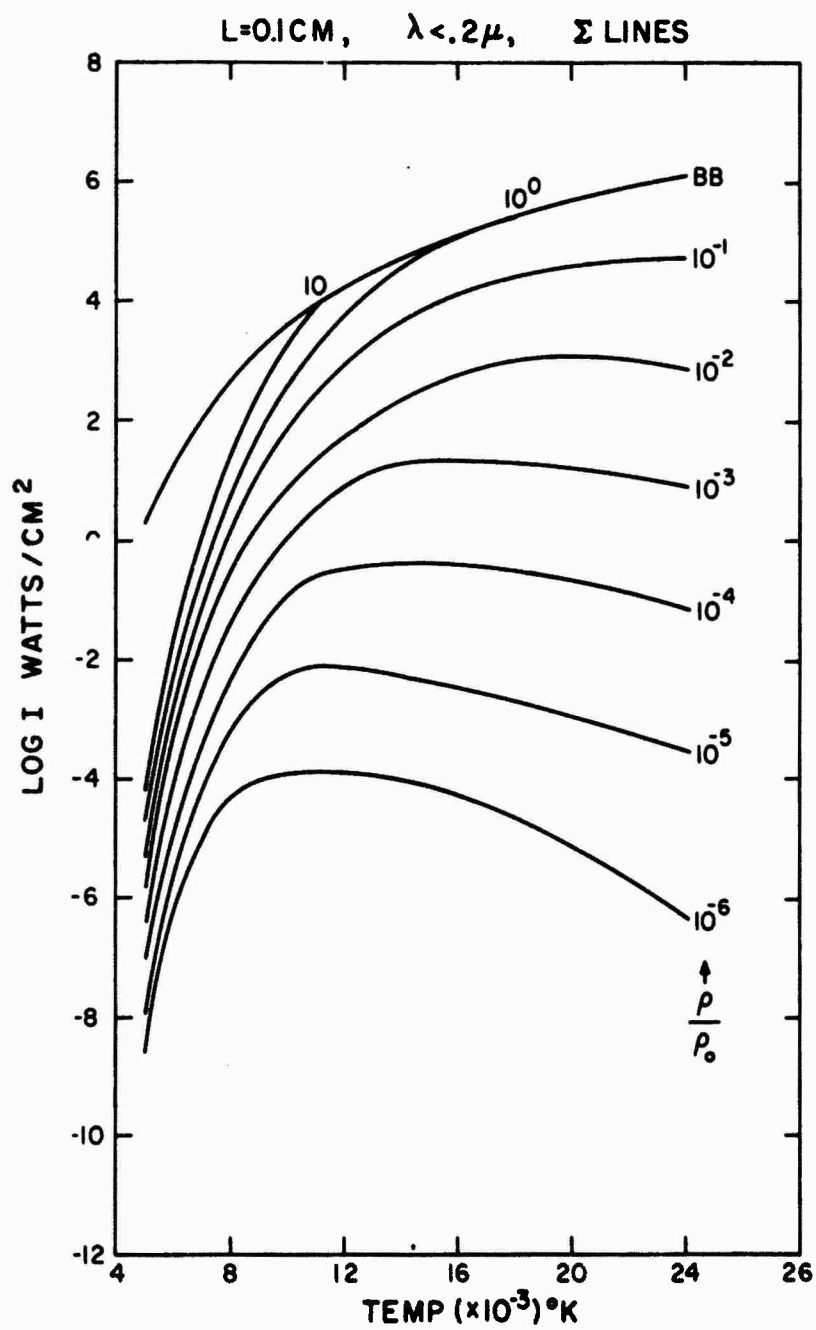


Fig. 9 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 0.1 cm thick for wavelengths less than 0.2μ .

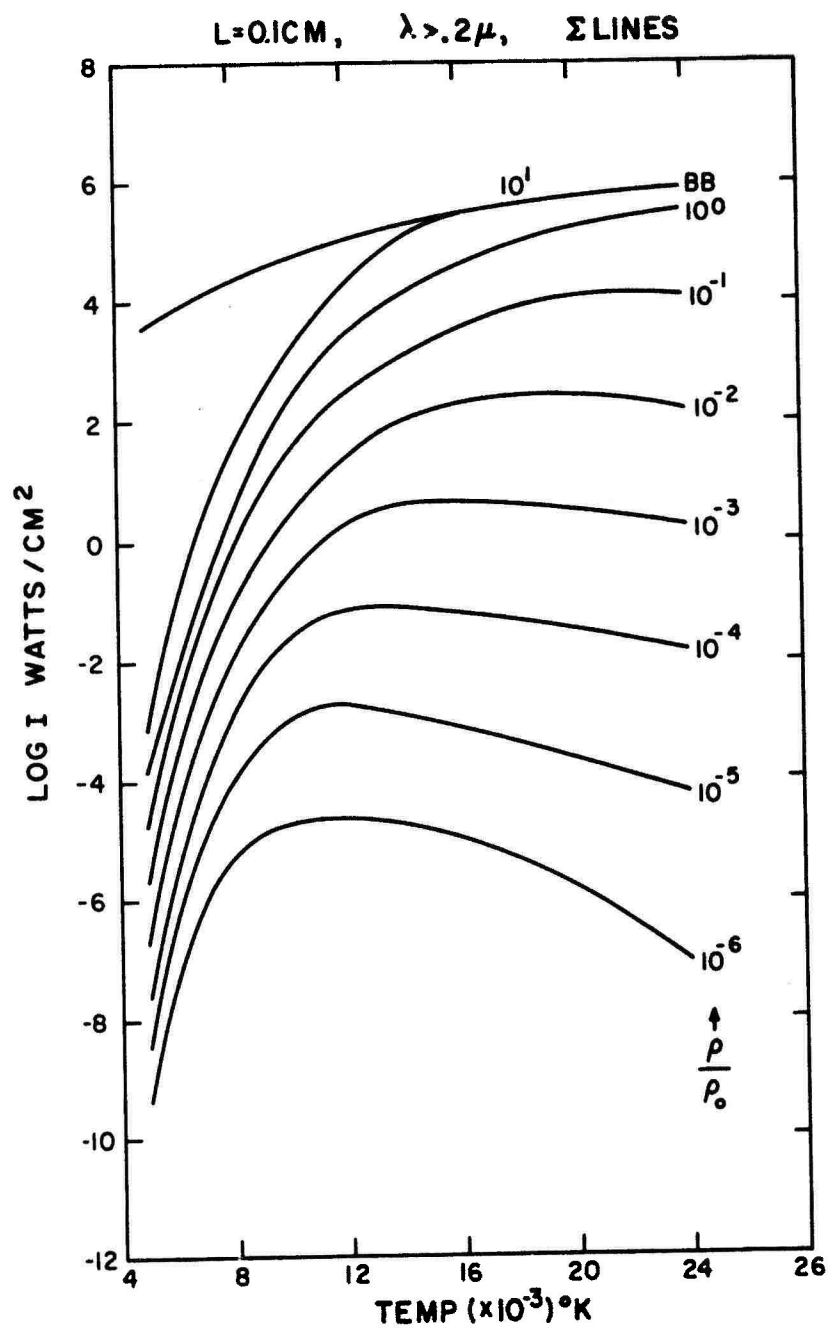


Fig. 10 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 0.1 cm thick for wavelengths longer than 0.2μ .

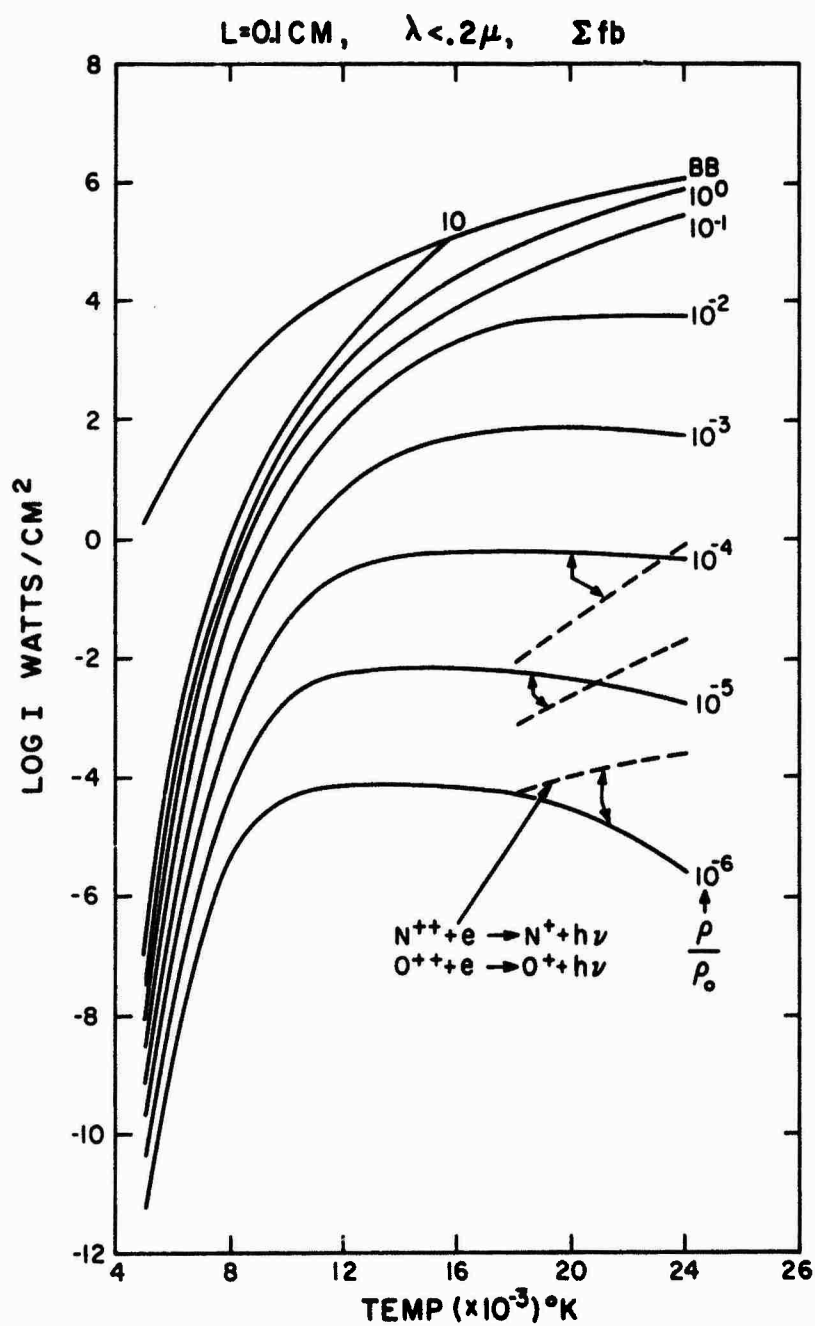


Fig. 11 Nitrogen and oxygen ion free bound radiation flux from one side of an infinite air slab 0.1 cm thick for wavelengths less than 0.2μ . The dashed lines are for the contributions from the N^{++} and O^{++} .

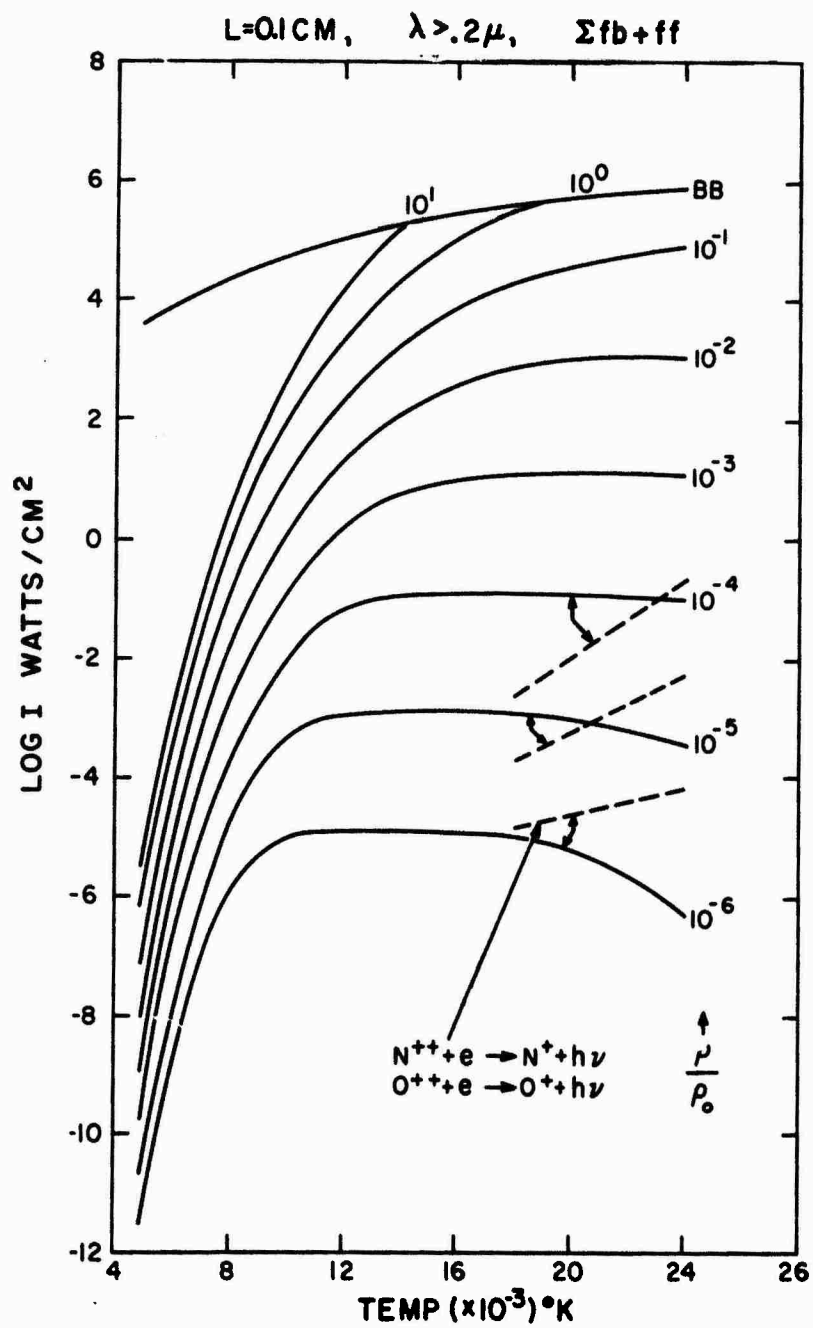


Fig. 12 Nitrogen and oxygen ion free bound and free free radiation flux from one side of an infinite air slab 0.1 cm thick for wavelengths greater than 0.2μ . The dashed lines are for the contribution from N^{++} and O^{++} .

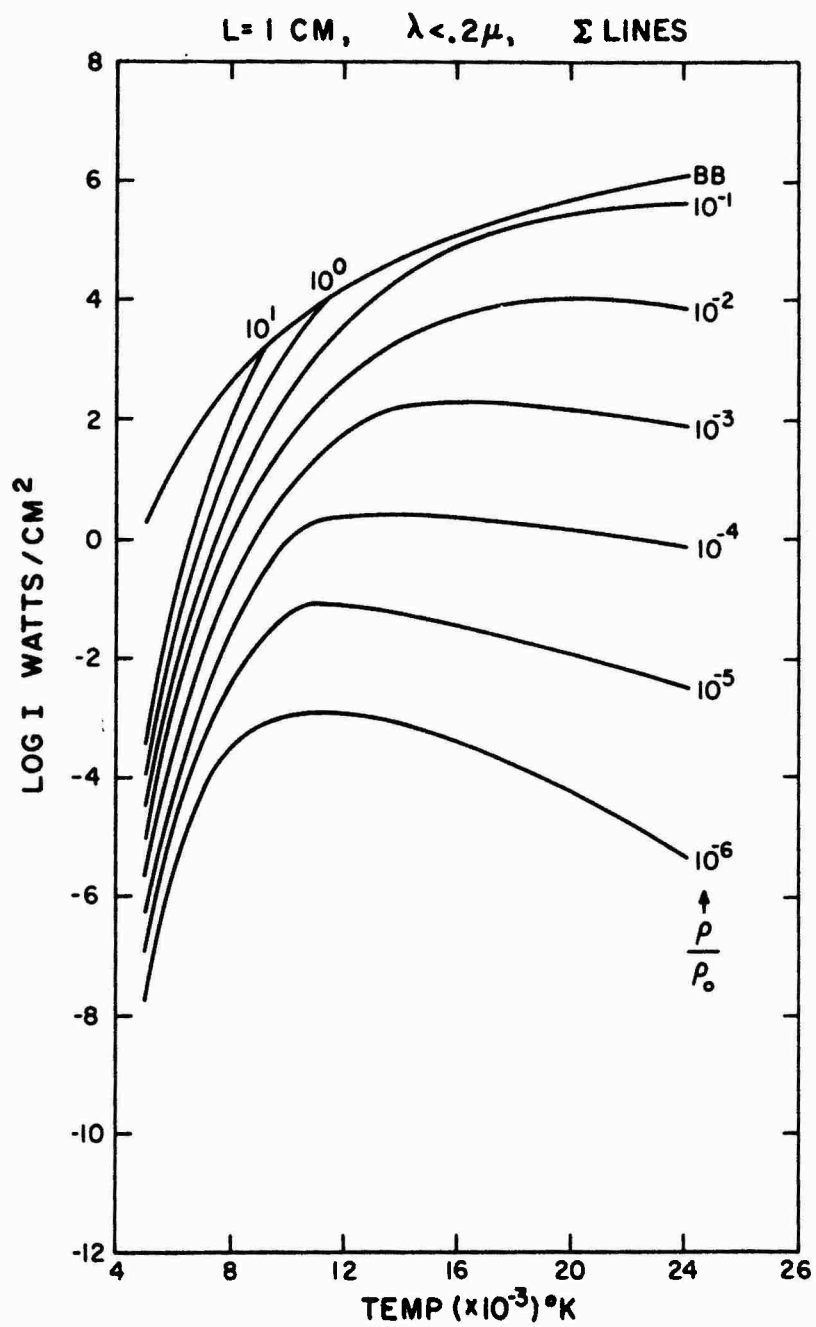


Fig. 13 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 1.0 cm thick for wavelengths less than 0.2μ .

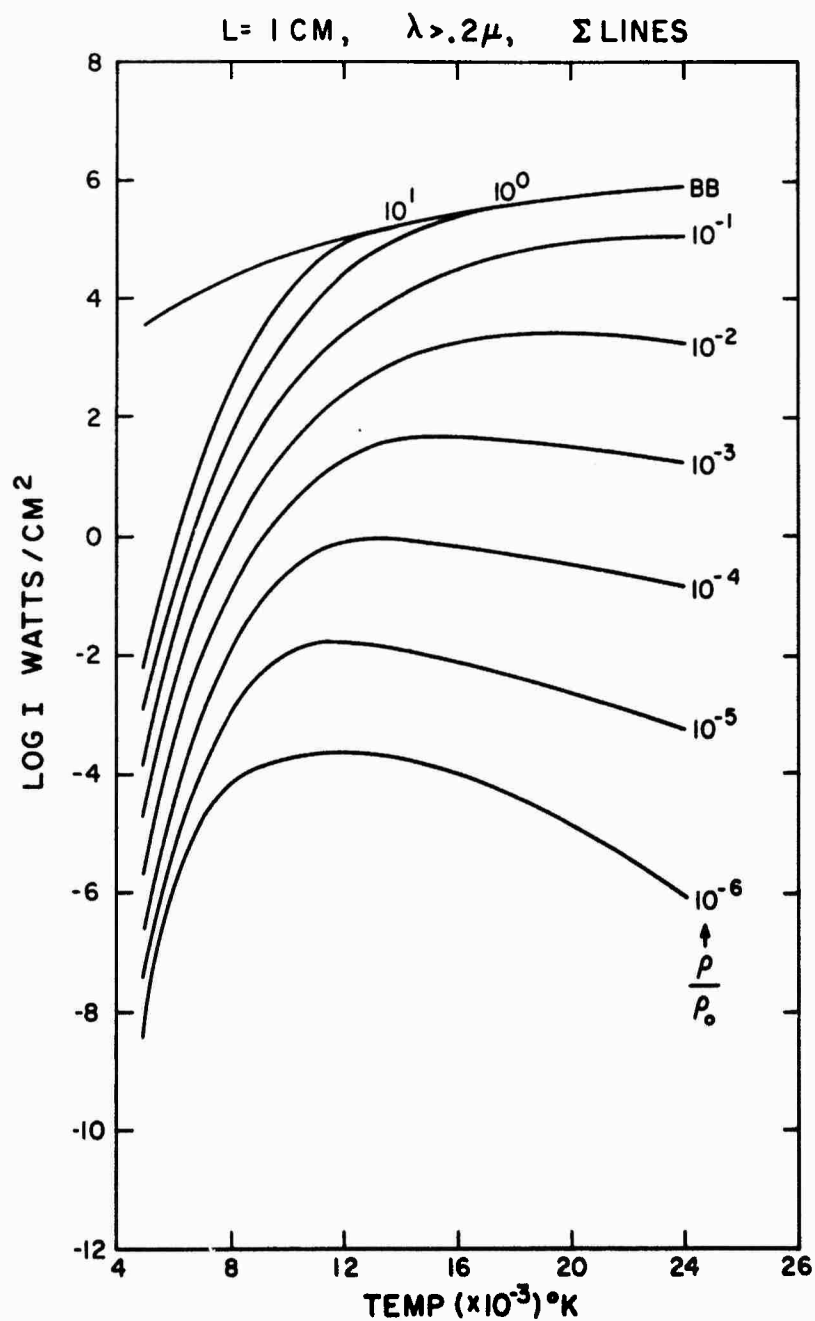


Fig. 14 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 1.0 cm thick for wavelengths longer than 0.2μ .

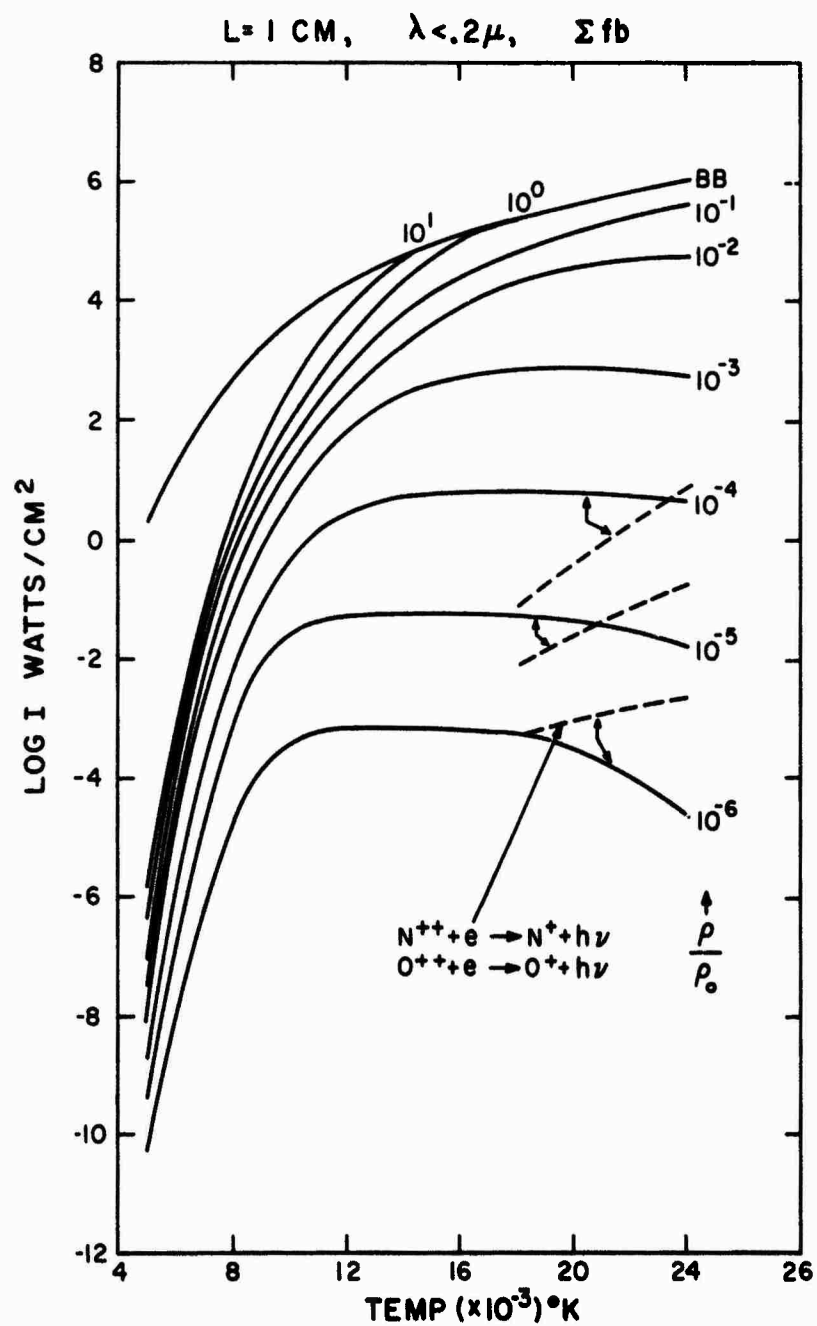


Fig. 15 Nitrogen and oxygen ion free bound radiation flux from one side of an infinite air slab 1.0 cm thick for wavelengths less than 0.2μ . The dashed lines are for the contributions from the N^{++} and O^{++} .

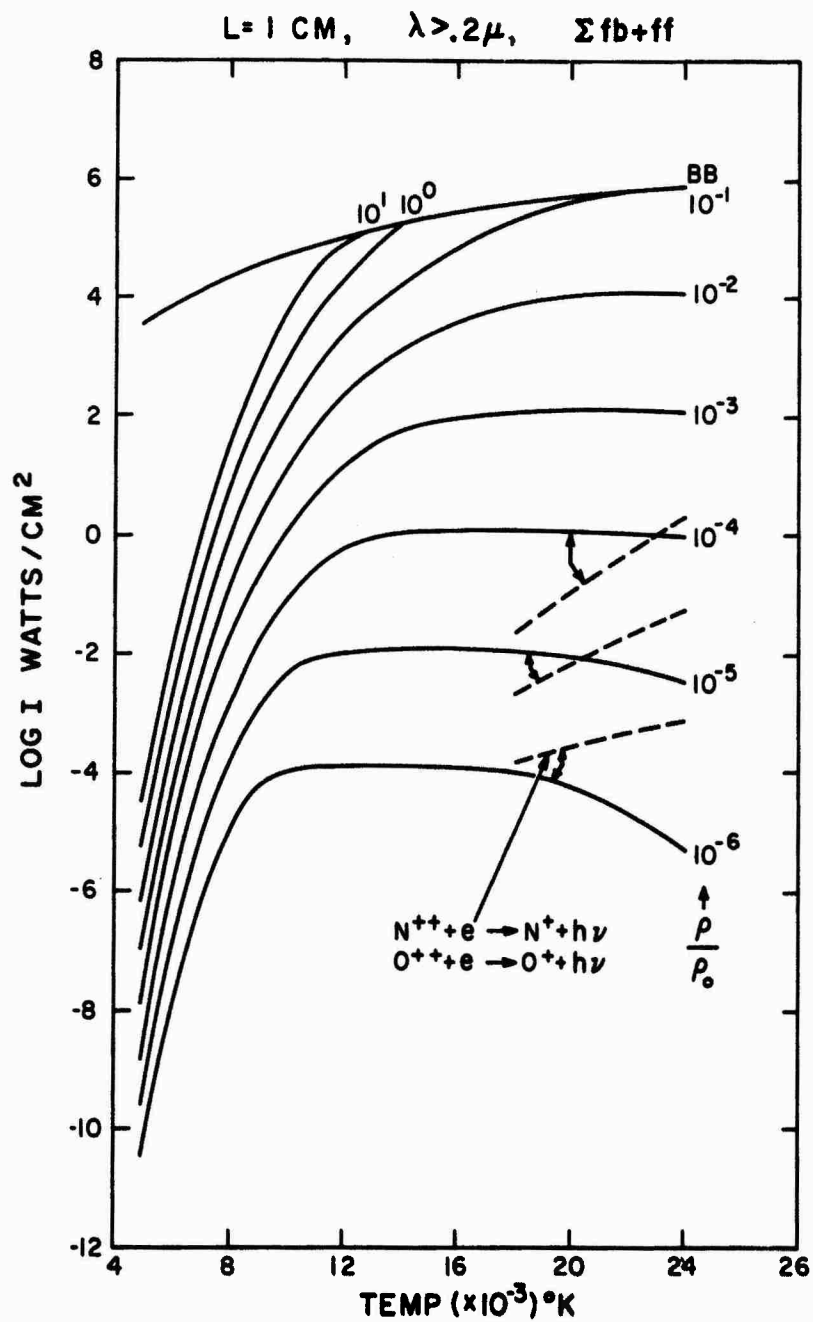


Fig. 16 Nitrogen and oxygen ion free bound and free free radiation flux from one side of an infinite air slab 1.0 cm thick for wavelengths greater than 0.2μ . The dashed lines are for the contribution from N^{++} and O^{++} .

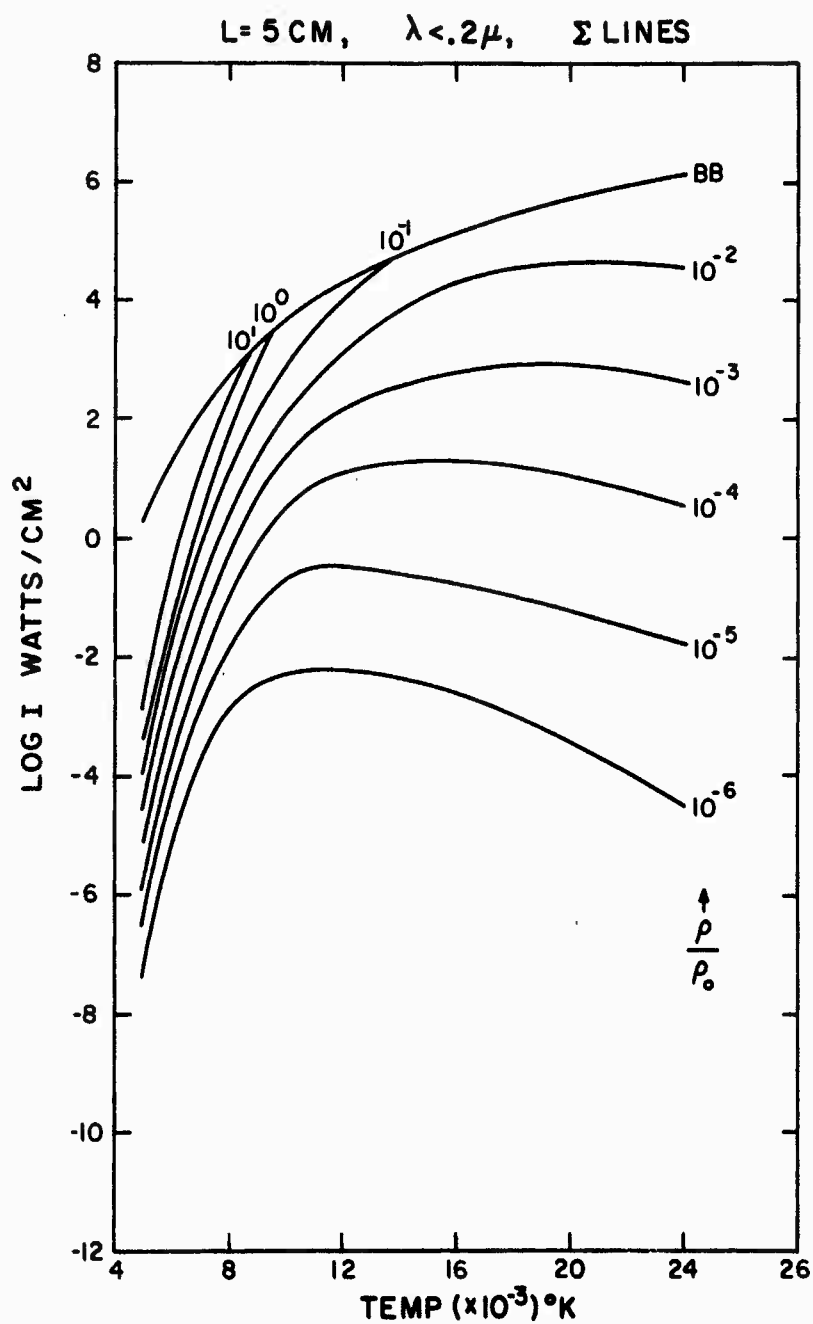


Fig. 17 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 5 cm thick for wavelengths less than 0.2μ .

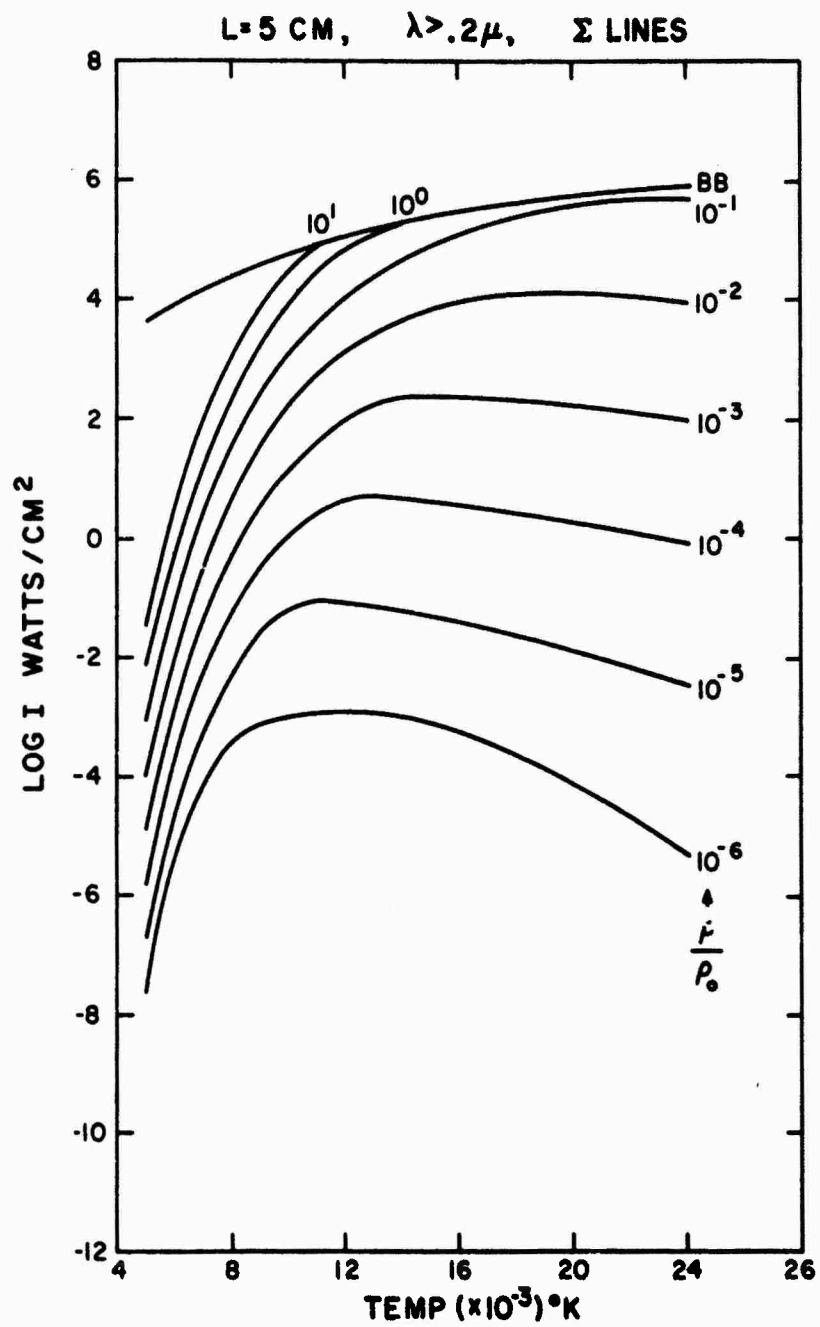


Fig. 18 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 5 cm thick for wavelengths longer than 0.2μ .

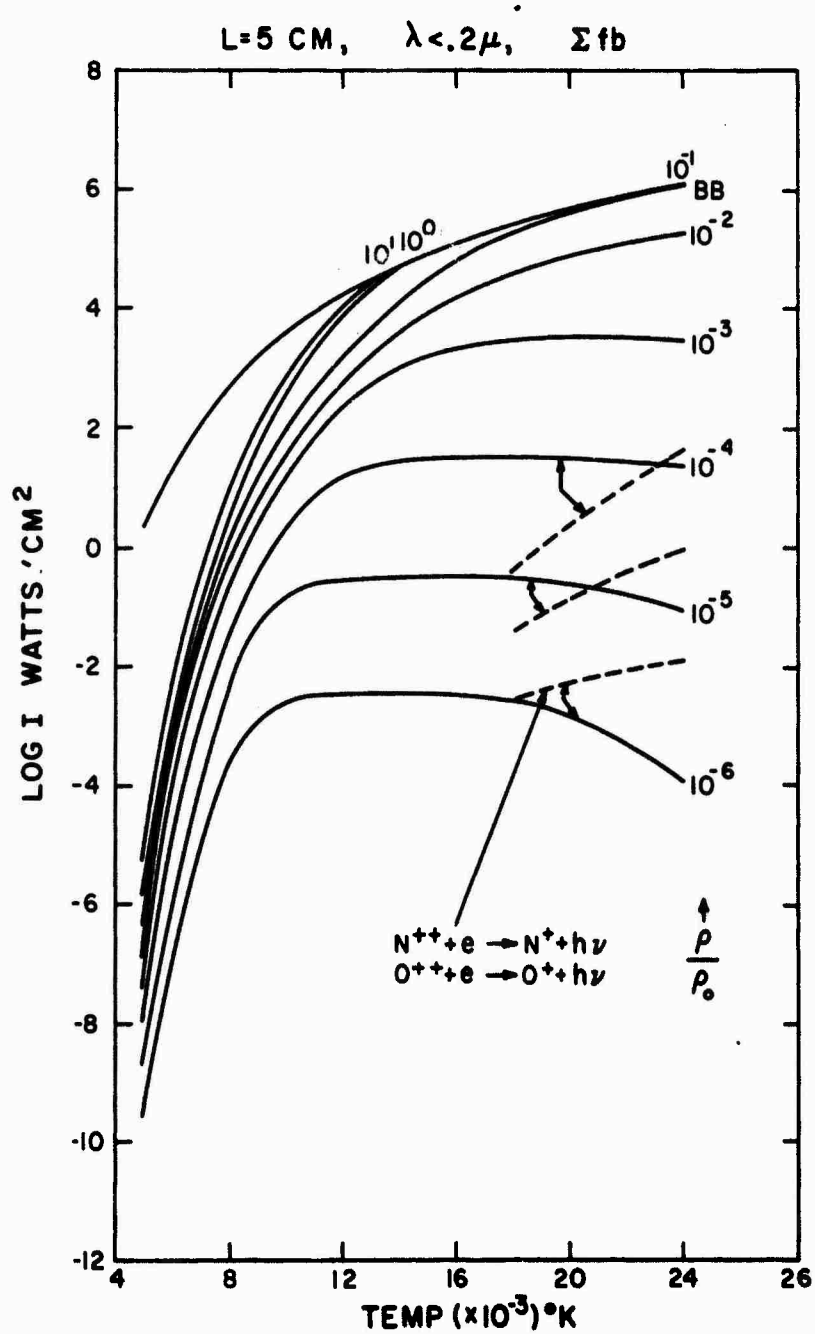


Fig. 19 Nitrogen and oxygen ion free bound radiation flux from one side of an infinite air slab 5 cm thick for wavelengths less than 0.2μ . The dashed lines are for the contributions from the N^{++} and O^{++} .

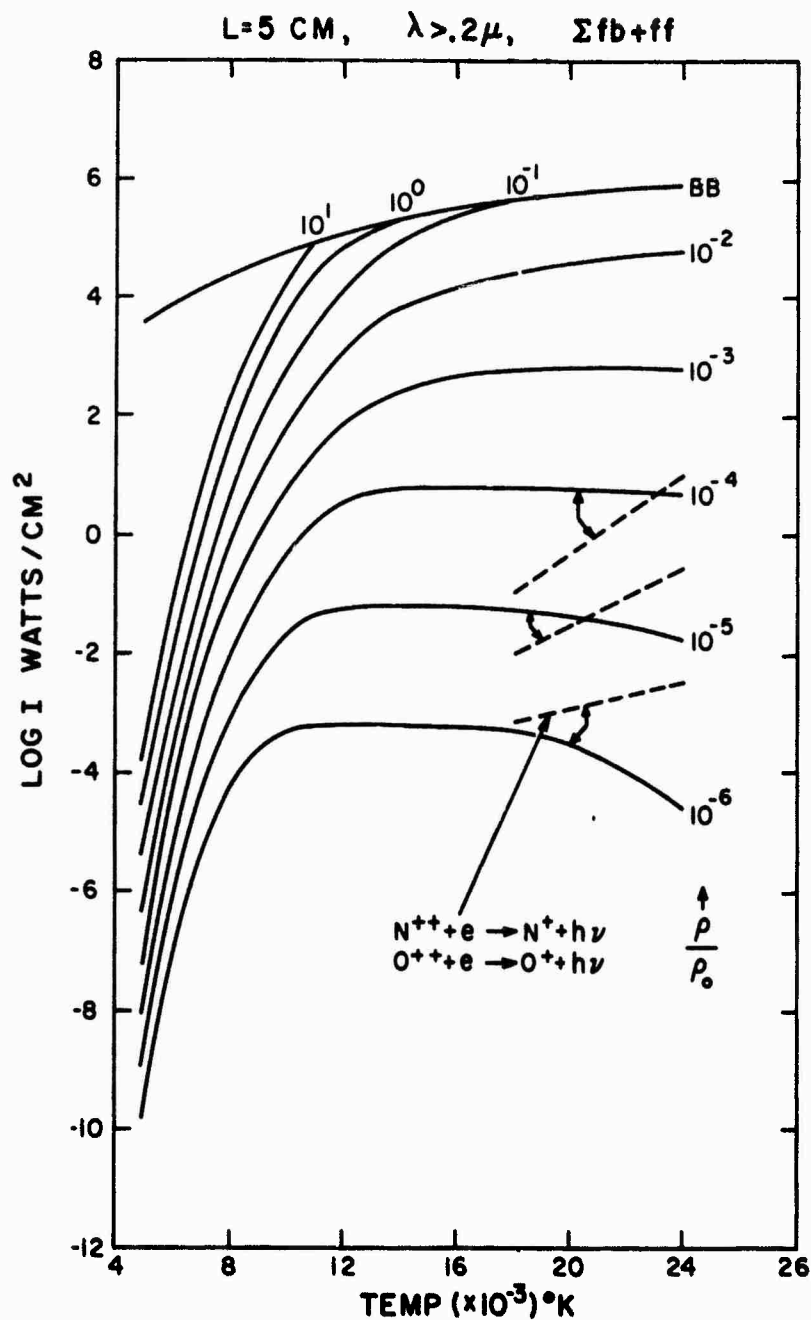


Fig. 20 Nitrogen and oxygen ion free bound and free free radiation flux from one side of an infinite air slab 5 cm thick for wavelengths greater than 0.2μ . The dashed lines are for the contribution from N^{++} and O^{++} .

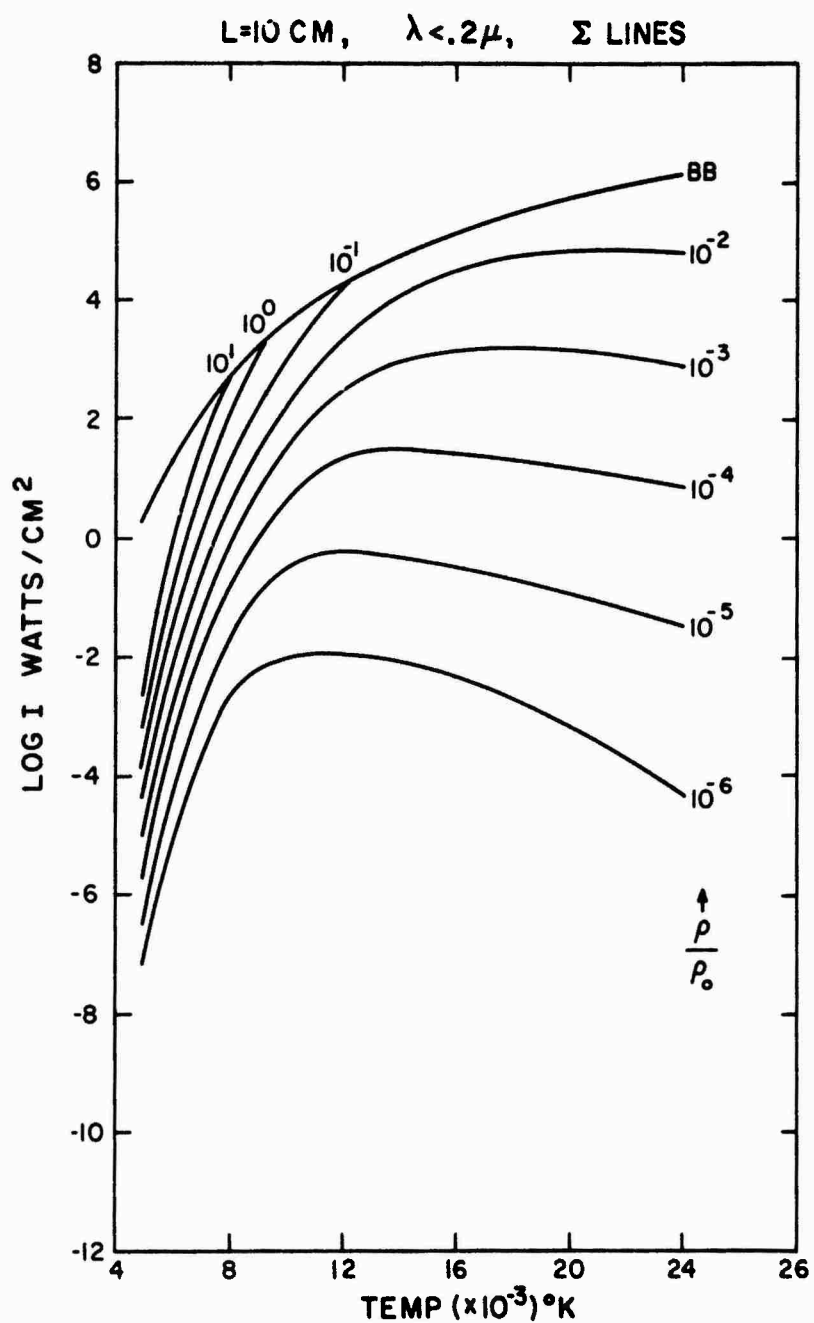


Fig. 21 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 10 cm thick for wavelengths less than 0.2μ .

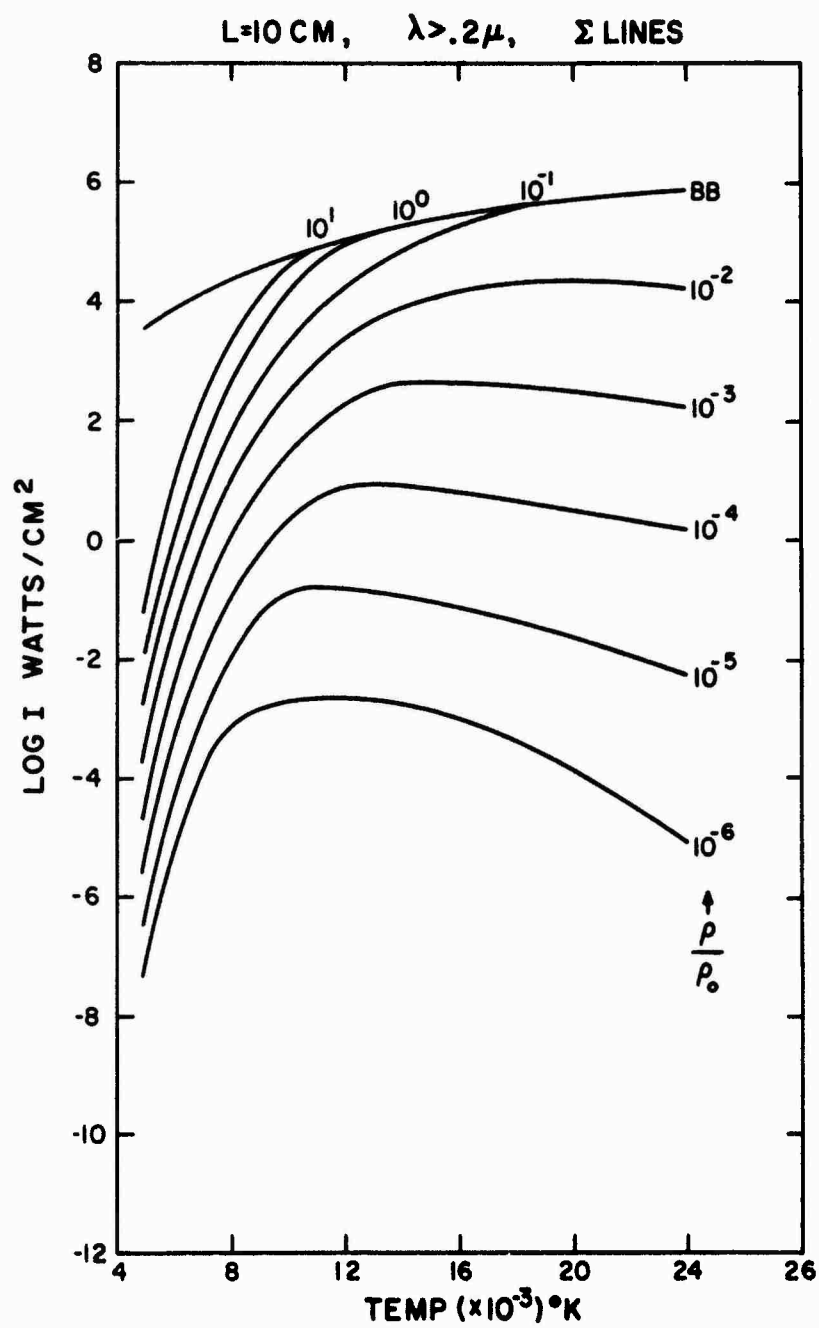


Fig. 22 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 10 cm thick for wavelengths longer than 0.2μ .

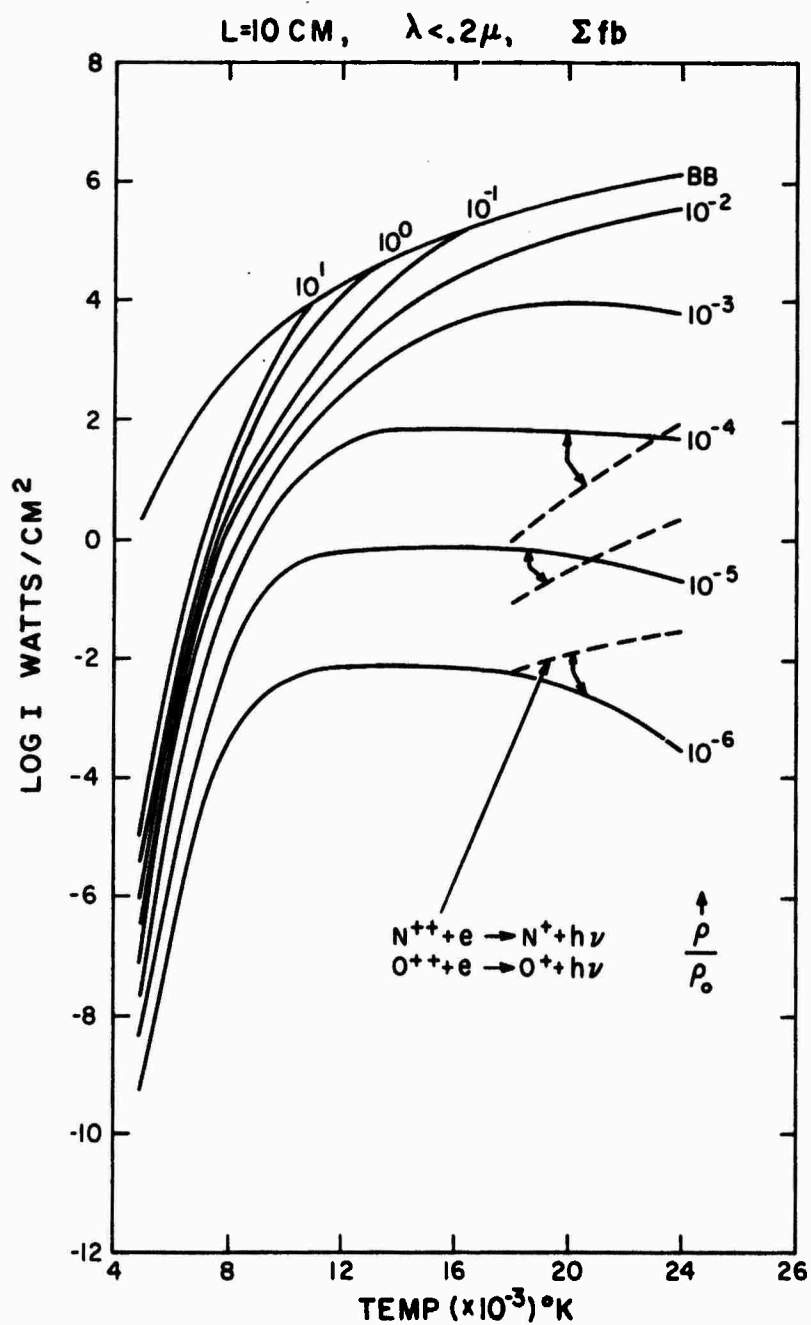


Fig. 23 Nitrogen and oxygen ion free bound radiation flux from one side of an infinite air slab 10 cm thick for wavelengths less than 0.2μ . The dashed lines are for the contributions from the N^{++} and O^{++} .

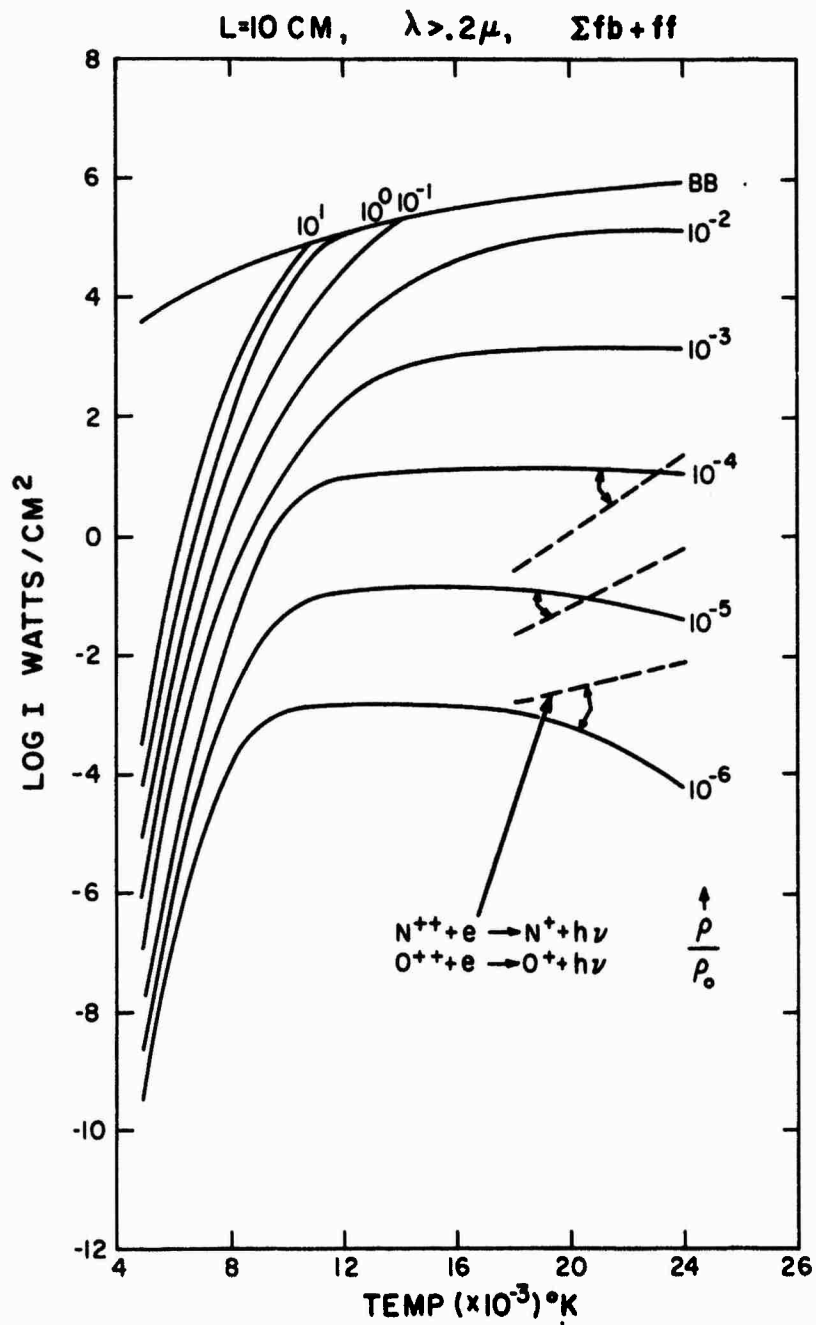


Fig. 24 Nitrogen and oxygen ion free bound and free free radiation flux from one side of an infinite air slab 10 cm thick for wavelengths greater than 0.2μ . The dashed lines are for the contribution from N^{++} and O^{++} .

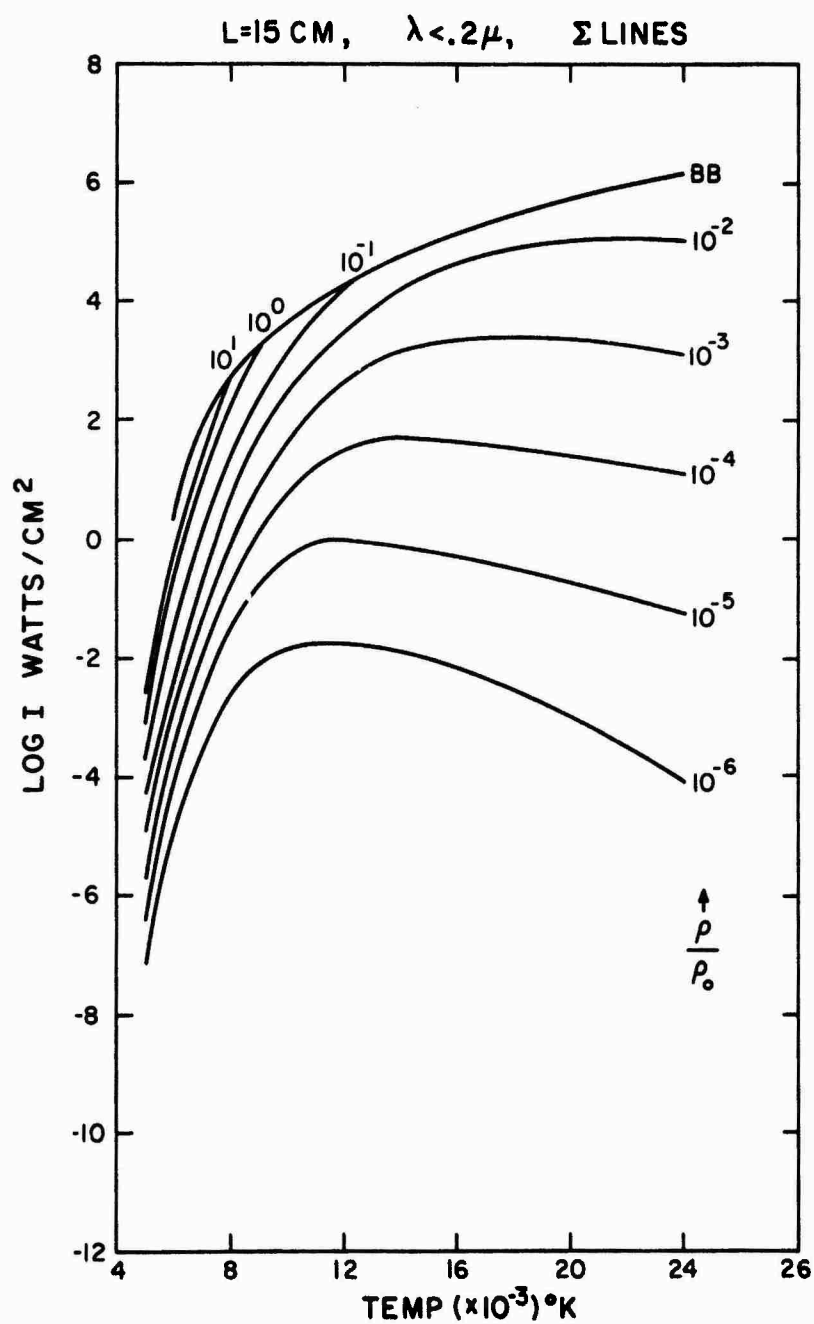


Fig. 25 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 15 cm thick for wavelengths less than 0.2μ .

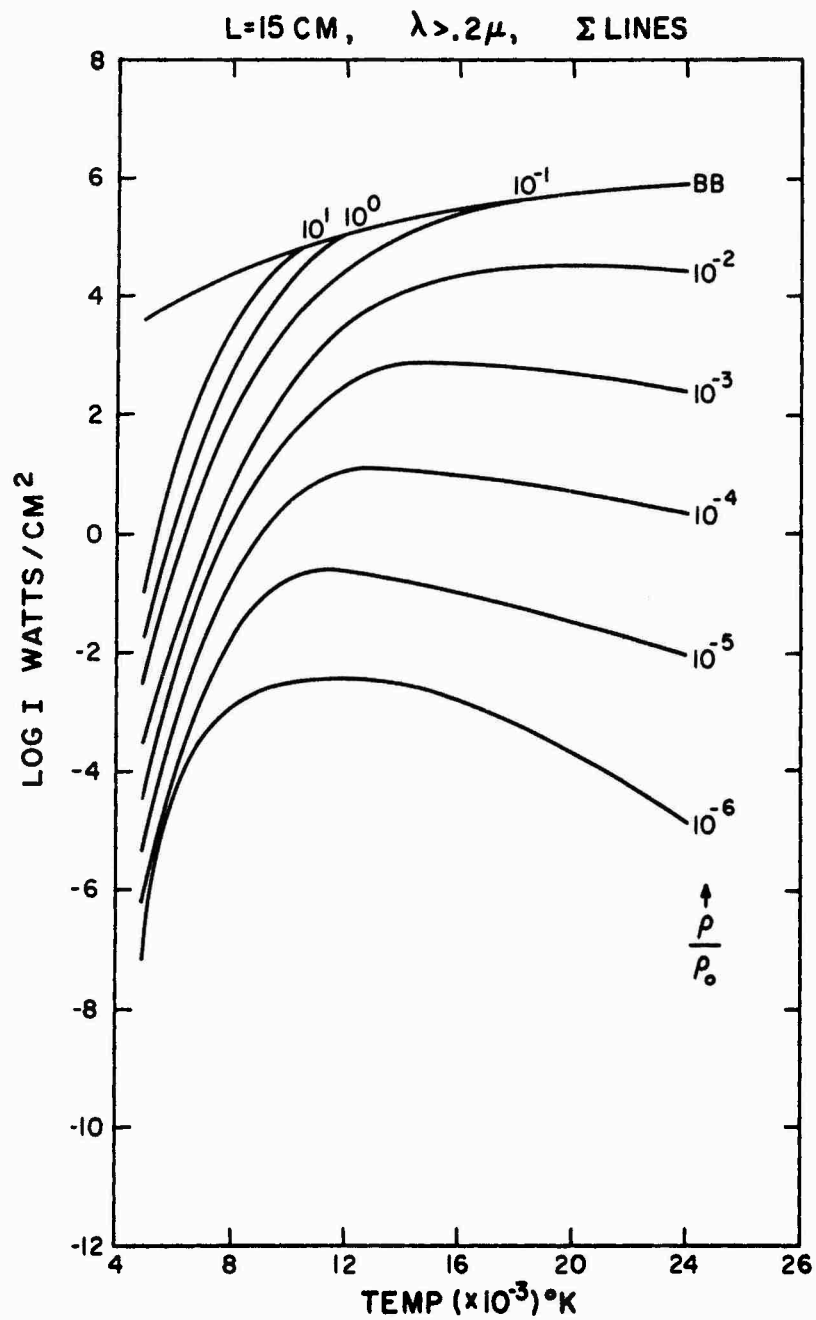


Fig. 26 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 15 cm thick for wavelengths longer than 0.2μ .

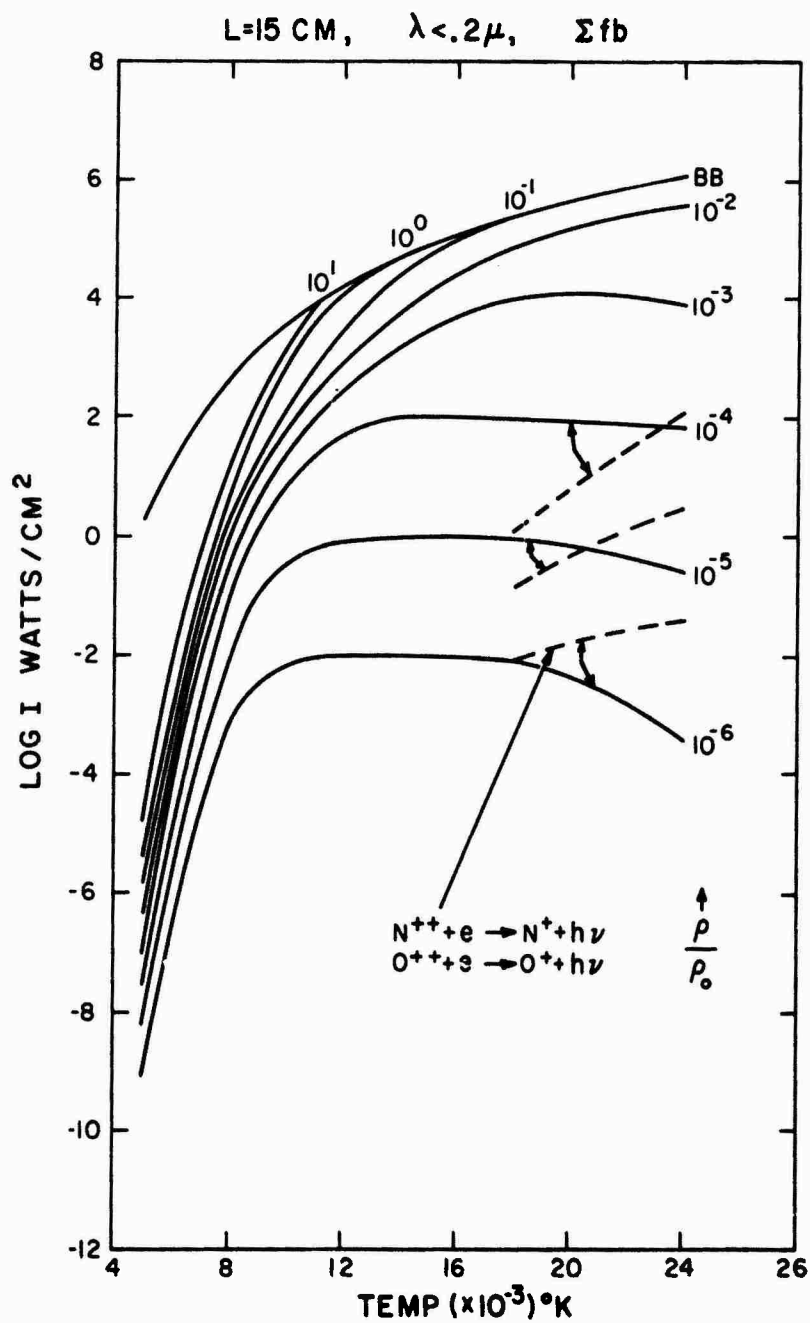


Fig. 27 Nitrogen and oxygen ion free bound radiation flux from one side of an infinite air slab 15 cm thick for wavelengths less than 0.2μ . The dashed lines are for the contributions from the N^{++} and O^{++} .

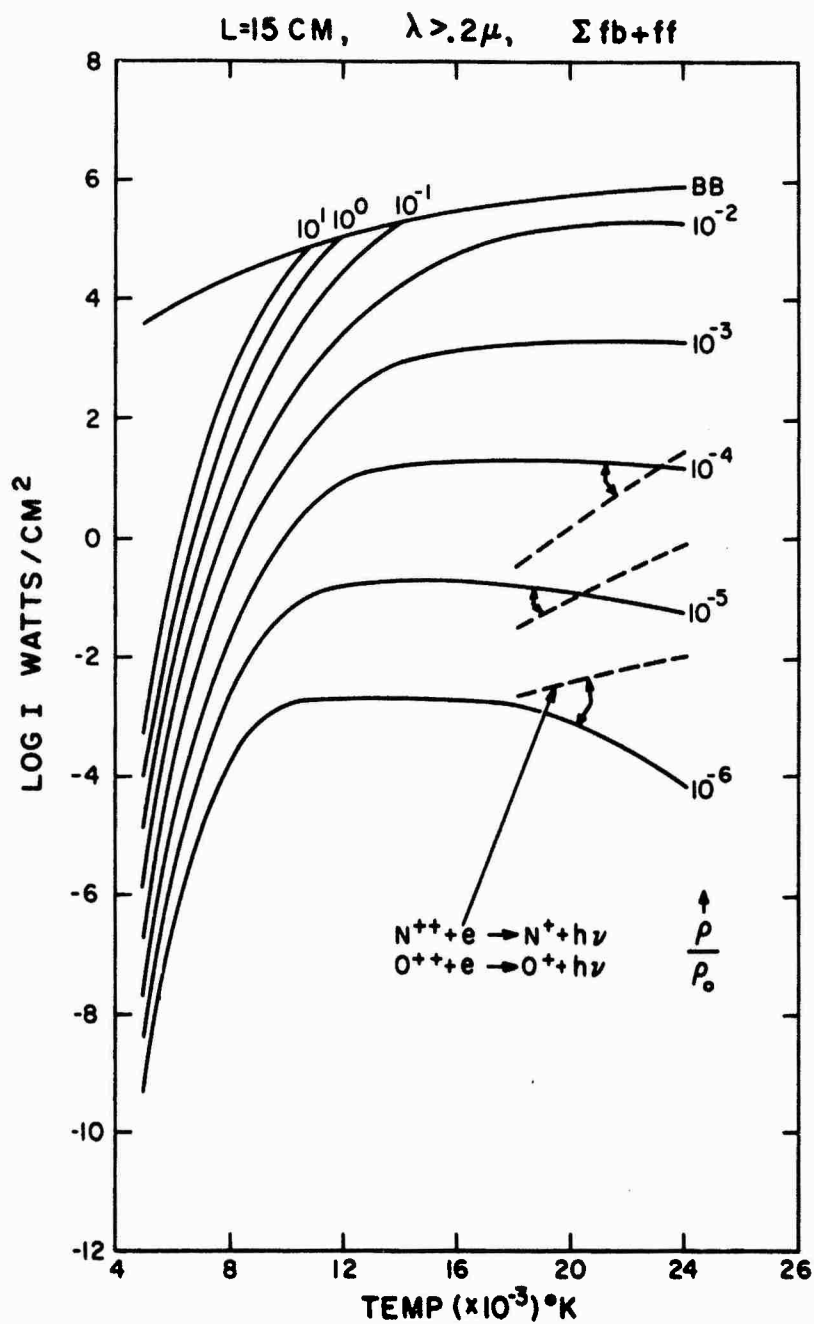


Fig. 28 Nitrogen and oxygen ion free bound and free free radiation flux from one side of an infinite air slab 15 cm thick for wavelengths greater than 0.2μ . The dashed lines are for the contribution from N^{++} and O^{++} .

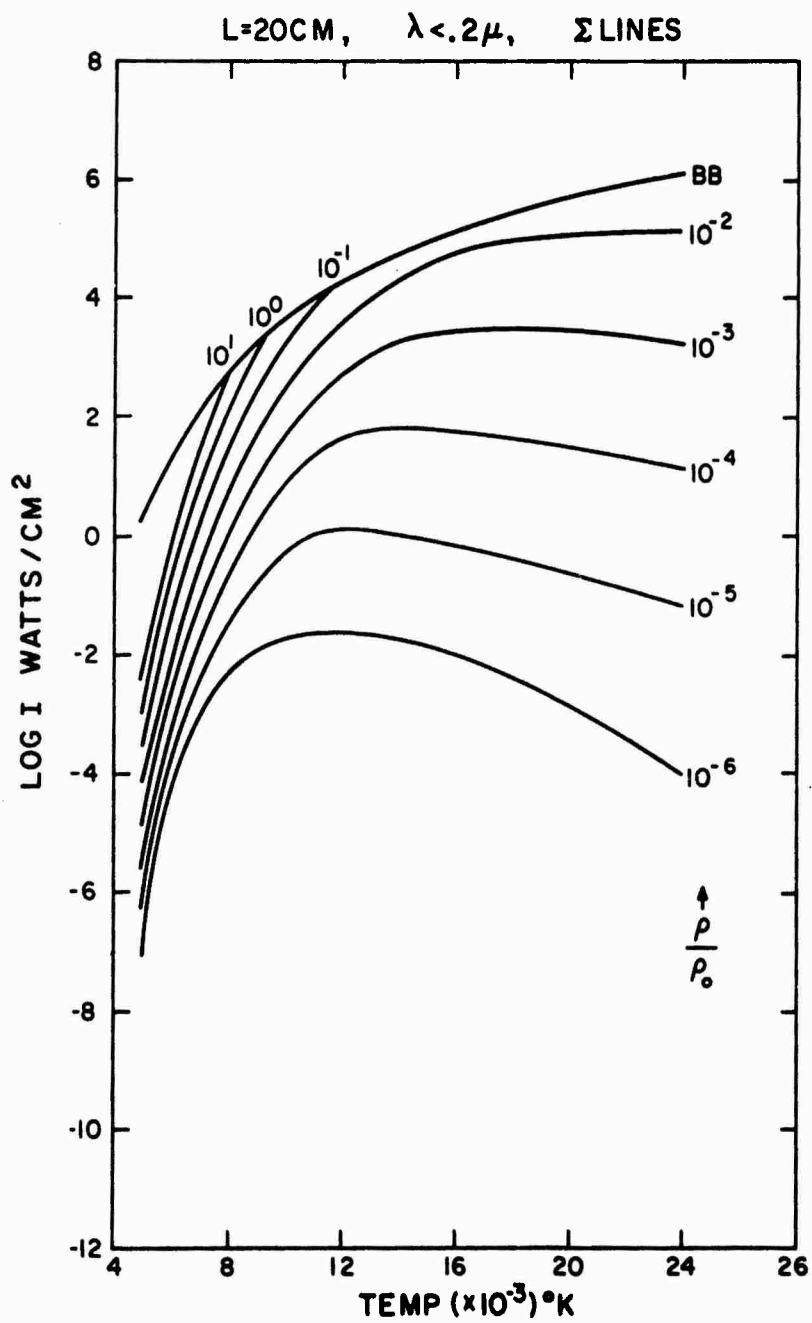


Fig. 29 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 20 cm thick for wavelengths less than 0.2μ .

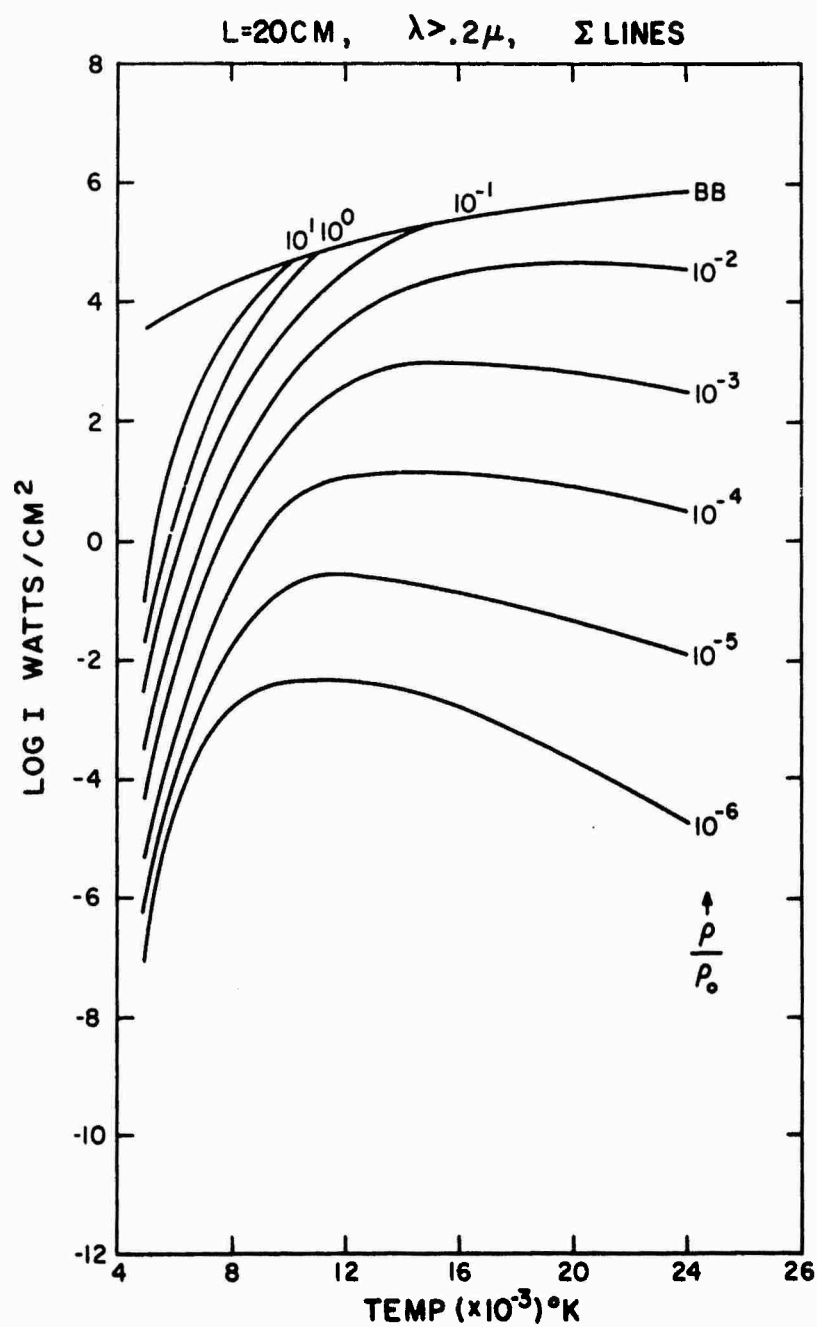


Fig. 30 Atomic nitrogen and oxygen line radiation flux from one side of an infinite air slab 20 cm thick for wavelengths longer than 0.2μ .

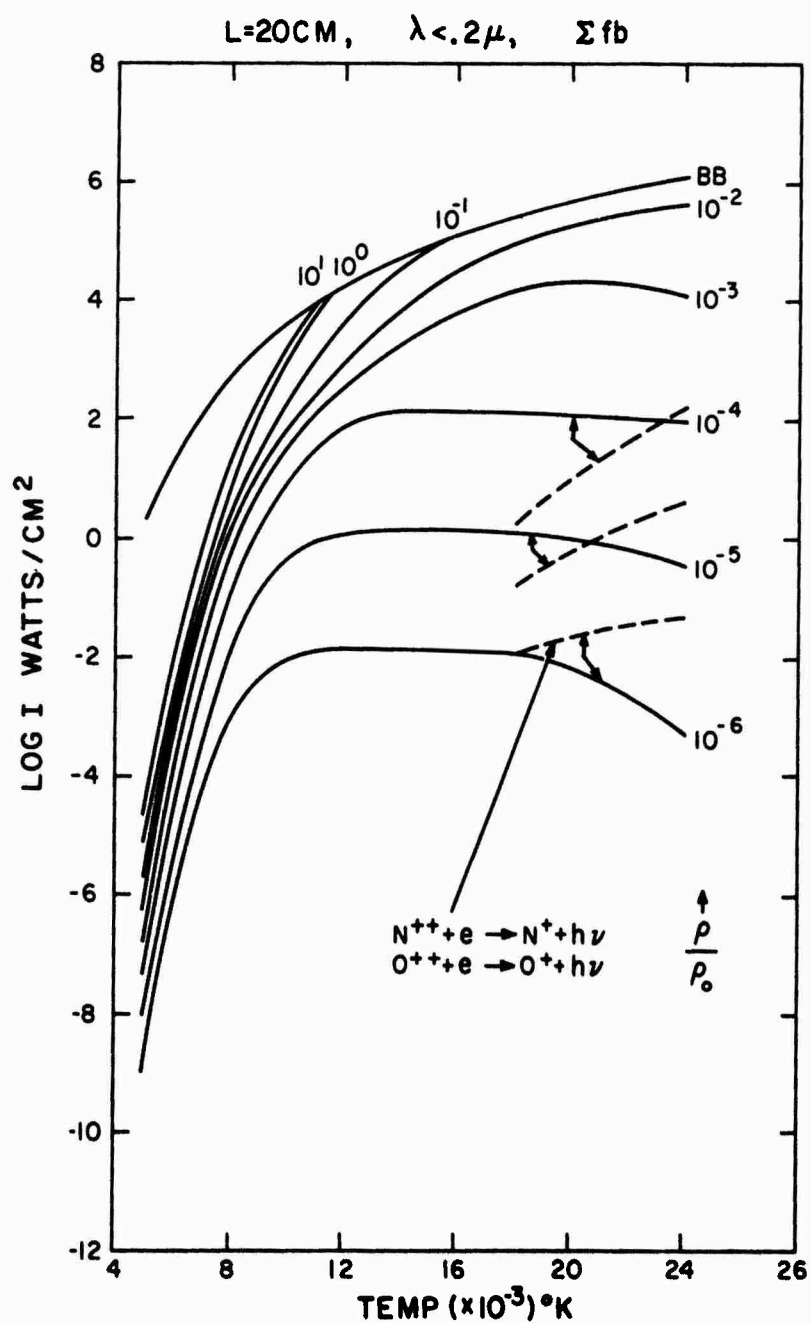


Fig. 31 Nitrogen and oxygen ion free bound radiation flux from one side of an infinite air slab 20 cm thick for wavelengths less than 0.2μ . The dashed lines are for the contributions from the N^{++} and O^{++} .

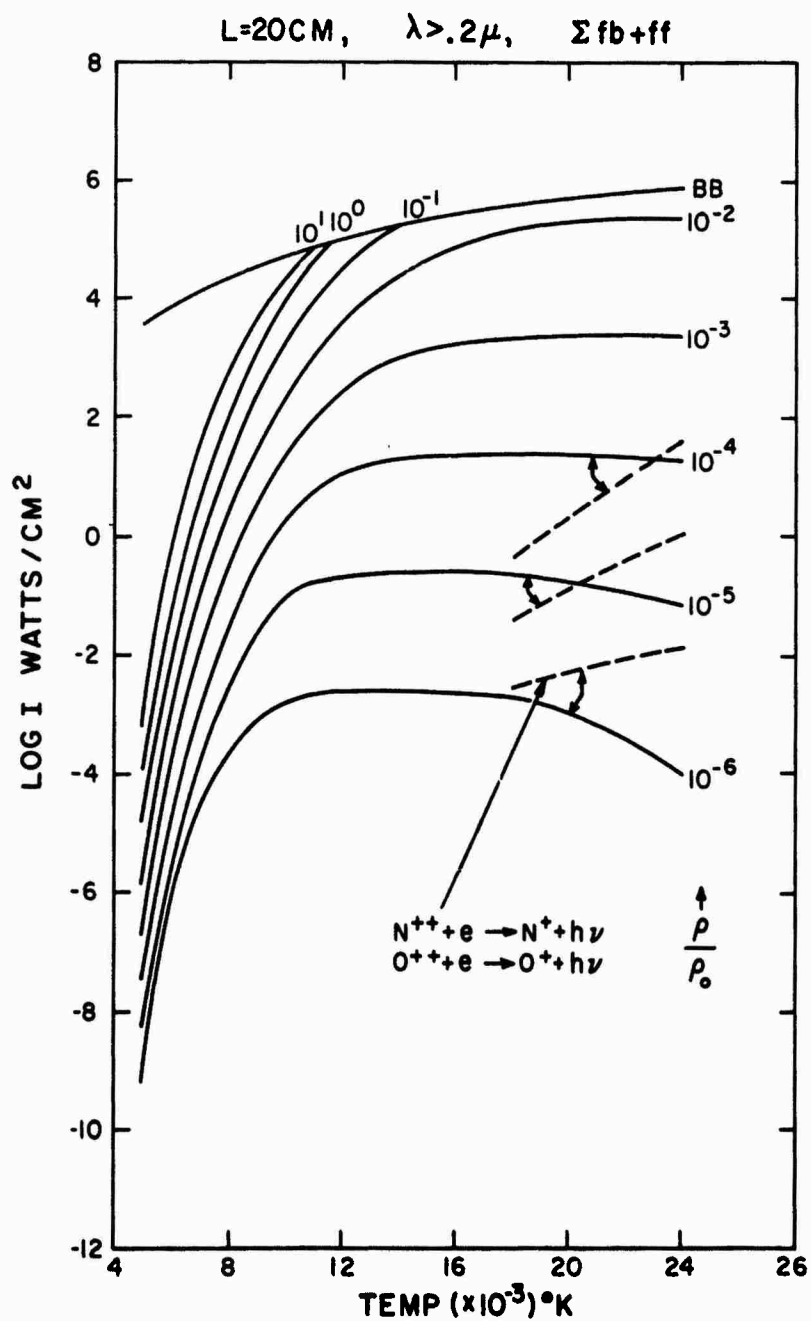


Fig. 32 Nitrogen and oxygen ion free bound and free free radiation flux from one side of an infinite air slab 20 cm thick for wavelengths greater than 0.2μ . The dashed lines are for the contribution from N^{++} and O^{++} .

B. N^{++} and O^{++} Radiation

At the higher temperatures and lower densities calculated for these radiation tables, the air is sufficiently doubly ionized such that the radiation produced by electrons being captured and decelerated by N^{++} and O^{++} becomes important. This radiation was calculated by use of expression (10) using an effective Z of 2 and carrying out the integration over wave-number to the vacuum ultraviolet photoabsorption edges of N .

C. O^- and N^- Continuum

The O^- continuum was calculated using the photoabsorption cross sections measured by Branscomb and Smith. Measurements of the photoabsorption cross section for N^- (1D) are still in question and calculations of the radiation from this source are not included.

D. Free-free Radiation from N^+ and O^+

The free-free contributions to the total radiation were calculated using the classical expression¹⁶

$$\frac{dI}{dV d\tilde{\nu} d\Omega} = 1.63 \times 10^{-35} T^{-1/2} N_e N_i Z^2 e^{-\frac{hc \tilde{\nu}}{kT}} \frac{W}{2 \text{ cm}^2 \text{ ster}} \quad (11)$$

Measurements obtained by Wilson¹⁷ in the infrared were used to infer an effective Z^2 of 1.5 for this source of radiation for air. The contributions from this source are included with the curves labelled 12 in figures 1 - 8.

E. Free-free Radiation from N_2 and N

Taylor has measured the effective Z^2 for scattering of electrons from neutral N_2 of .022 and from N of .009. These values are used in conjunction with expression (11) to predict the curves labelled 16 and 17 respectively.

ATOMIC LINE RADIATION

The total intensity of an optically thin spectral line emitted from one side of an infinite slab of thickness L is

$$I = \left[N_{n,\ell} \right] L 2 \pi^2 f r_o BB_{\nu_1} \left(1 - e^{-\frac{hc \tilde{\nu}}{kT}} \right) \quad (12)$$

where $N_{n,\ell}$ is the number of atoms per unit volume in the absorbing state for the transition, f is the absorption bound-bound oscillator strength, r_o is the classical electron radius, BB_{ν_1} is the black body intensity perpendicular to the surface and $(1 - e^{-hc \tilde{\nu}/kT})$ is the induced emission term.

In many situations, these lines undergo a certain amount of self-absorption which reduces the total measured intensity from the completely optically thin situation. In order to calculate the self-absorption, one must know the principle broadening mechanisms in the gas.²¹ The line width and shape are the pertinent parameters which must be known. Lines also undergo a small wavelength shift for certain types of broadening, however, for the purposes of making these radiation tables this shift has been disregarded since it has little effect on the results. For the densities and temperatures considered in these calculations, doppler and electron broadening mechanisms dominate over natural and pressure broadening by neutrals.

For the purposes of making calculations for these radiation tables, the electron impact formula of Armstrong²¹ and Stewart and Pyatt²³ was used. The total line width in wave numbers is given by

$$W_{\tilde{\nu}} = 5.7 \times 10^{-16} N_e \frac{n^4}{Z^2} T^{-1/2} \text{ cm}^{-1} \quad (13)$$

where N_e is the electron density, n is the principle quantum number of the

upper state, Z is the total charge of the atom minus one and T is the temperature. Self-absorption of Lorentz, or dispersion shaped, non-overlapping lines, is treated by various authors^{20, 24, 25}. For our purposes, we followed the definition of Penner²⁴ and Plass²⁵ for $f(x)$ where

$$x = \frac{[N_{ne}] f r_o L}{W_{\nu}} = \frac{N_{n,l} f r_o Z^2 T^{1/2}}{5.7 \times 10^{-16} N_T n^4} \cdot \frac{N_T}{N_e} L = K \frac{N_T}{N_e} L \quad (14)$$

$f(x)$ versus $K \frac{N_T}{N_e} L$ was modified according to the work of Olfe²⁰ so as to be able to treat self-absorption for line radiation from one side of an infinite slab.

The line transitions listed by Griem²⁶ along with their f numbers includes most of the significant transition in N and O and were the ones used to set up a computer program to calculate the line contributions to the total radiation in two spectral regions, divided at 2000 Å and also carried out with slab thickness as a parameter. The assumption was made that all lines were non overlapping and also that the contribution from the lines not listed in Griem were not significant contributors to the total radiation.

The line widths as predicted by expression (13) were larger than doppler widths except at low temperatures and at low densities both regions where line radiation was not an important contribution or at a density where self-absorption effects were not large. Therefore only electron impact broadening was considered.

The calculations of the line radiation especially in the ultraviolet are open to the criticism that not enough self-absorption was included. The reason for this is that the electron impact widths as predicted by expression (13) are too large for the nitrogen line transitions occurring in the ultraviolet and visible regions of the spectrum. Stampa²⁷ has made nitrogen line width measurements on arcs which show that the approximation formula used in these calculations over predicts the line widths by a factor from approximately 4 to 9. Neglecting the doppler shape, this means that for

situations where significant self-absorption is taking place, namely in the vacuum ultraviolet, the present radiation calculations would over predict the radiation by a factor ranging from 2 to 3. The radiation emitted from one side of infinite gas slab from a strongly self-absorbed Lorentz shaped line varies as the square root of the half width. Since doppler broadening is present, the over estimation is not as bad as a factor of 2 to 3 and for the purposes of making engineering numbers for radiation heating the present radiation calculations are reasonable.

CONCLUSION

These tables have been designed primarily for making radiation heating estimates for re-entry into the earth's atmosphere, and consequently a conservative approach has been made commensurate with a fairly good understanding of the radiation physics. In the region where radiation heating is generally important free-bound and atomic line radiation are the important radiating mechanisms. Self-absorption of these two sources of radiation play a significant role especially in the vacuum ultraviolet and must be included in realistic radiation calculations.

An infinite slab geometry has been used throughout since this is a geometry which lends itself well to making self-absorption calculations and is also one which reasonably simulates re-entry flight geometries. The error produced by the use of an infinite slab is in a conservative direction by about 10 - 15%. The line broadening formula used for the line radiation enables a relatively easy approach to calculating the fluxes from the lines although the use of it also tends to overpredict the radiation in the ultraviolet. The formulas developed for the free bound radiation are hydrogen like for transitions in the visible and is a model which does fit reasonably well with shock tube and arc measurements. In order to account for non-hydrogen like behavior of transitions involving ground states of N we have used the calculated and measured value of 10^{-17} cm^2 with no wavelength dependence.

For the comparison of air data and calculations based on the methods used in this study, attention should be given to the references in Table I and in general, the calculations embody all the oscillator strengths and cross sections which we feel give the best phenomenological fit to air data.

ACKNOWLEDGMENT

Credit is given to the many members of the Avco-Everett Research Laboratory, especially, A. Fairbairn, J. C. Keck, B. Kivel, R. Taylor, K. Wray, and L. Young, who gave many valuable suggestions and to B. Kaplan, A. Textoris, and T. Tsika who aided in much of the compilation of the results.

REFERENCES

1. Wick, B. H., "Radiative Heating of Vehicles Entering the Earth's Atmosphere," presented to the Fluid Mechanics Panel of Advisory Group for Aeronautical Research and Development, Brussels, Belgium, April 3 - 6, 1962.
2. Allen, R. A., "Air Radiation Tables: Spectral Distribution Functions for Molecular Band Systems," Avco-Everett Research Laboratory Research Note 560, to be published.
3. Keck, J., Camm, J., Kivel, B. and Wentink, T., Jr., "Radiation from Hot Air" Part II, Avco-Everett Research Laboratory Research Report 42, *Annals of Phys.*, 7, 1-38 (May, 1959).
4. Patch, R. W., Shackleford, W. L. and Penner, S. S., "Approximate Spectral Absorption Coefficient Calculations for Electronic Band Systems belonging to Diatomic Molecules", *J. Quant. Spect. Rad. Transf.* 2, 263-271 (1962).
5. Keck, J. C., Allen, R. A., and Taylor, R. L., "Electronic Transition Moments for Air Molecules", Avco-Everett Research Laboratory Research Report 149, *J. Quant. Spect. Rad. Transf.* 3, 335-353 (Oct.-Dec. 1963).
6. Milliken, R. S., "Intensities of Electronic Transitions in Molecular Spectra", *J. Chem. Phys.* 7, 14-21 (January, 1939).
7. Nicholls, R. W., "Transition Probabilities of Aeronomically Important Spectra", *Annals of Phys.*, 20 144-181 (April-June, 1964).
8. Herzberg, G., "Molecular Spectra and Molecular Structure, Vol. I Spectra of Diatomic Molecules" (Van Nostrand Co., Inc., New York 1950).
9. Allen, R. A., "Nonequilibrium Shock Front Rotational, Vibrational and Electronic Temperature Measurements", Avco-Everett Research Laboratory Research Report 186, *J. Quant. Spect. Rad. Transf.* 5, 511, (1965).
10. Fairbairn, A. R., "Shock Tube Study of the Oscillator Strength of the C₂ Swan Band", Avco-Everett Research Laboratory Research Note 557, (July, 1965).
11. Hilsenrath, J. and Klein, M., "Tables of Thermodynamic Properties of Air in Chemical Equilibrium including Second Virial Corrections from 1500°K to 15,000°K", AEDC-TR-65-58 (March, 1965).

12. Gilmore, F. R., "Equilibrium Composition and Thermodynamic Properties of Air to 24,000°K, Rand Corp. RM-1543 (Aug. 1955).
13. Bethe, H. A. and Salpeter, E. E., Quantum Mechanics of One and Two Electron Atoms, New York, Academic Press (1957).
14. Moore, C., Atomic Energy Levels, Vol. I NBS Circ. 467, (June 15, 1949).
15. Aller, L. H., Astrophysics: The Atmospheres of the Sun and Stars. New York, Ronald Press, 1953 pp 122-180.
16. Unsold, A., Ann. Physik 33, 607 (1938).
17. Allen, R. A., Textoris, A., and Wilson, J., "Measurements of the Free-Bound and Free-Free Continua of Nitrogen, Oxygen and Air", Avco-Everett Research Laboratory Research Report 195, J. Quant. Spectr. Rad. Transf. 5, 95-108 (1965).
18. Bates, D. R. and Seaton, J. J., "The Quantal Theory of Continuous Absorption of Radiation by Various Atoms in their Ground States II. Further Calculations on Oxygen, Nitrogen and Carbon, Monthly Not. Roy. Astro. Soc. 109, No. 6, 698-704 (1949).
19. Ehler, A. W., and Weissler, G. L. (1955) F. Opt. Soc. Amer. 45, 1035.
20. Olfe, D. B., "Mean Beam Length Calculations for Radiation from Non-Transparent Gases", J. Quant. Spect. Rad. Transf. 1, 169-176.
21. Baranger, M., "Spectral Line Broadening in Plasmas," Vol. 13, Atomic and Molecular Processes, Academic Press, New York (1962), 493-548.
22. Armstrong, B. H., "Broadening of Balmer Lines for High Quantum Number", J. Quant. Spect. Rad. Transf. 4, (May-June 1964).
23. Stewart, J. C. and Pyatt, K. D., Jr., "Theoretical Study of Optical Properties. Photon Absorption Coefficients, Opacities and Equations of State of Light Elements, including the Effect of Lines", Final Report Contract AF 29(601)-2807, General Atomic Division, General Dynamics Corporation (September 1961).
24. Penner, S. S., Qualitative Molecular Spectroscopy and Gas Emissivities, (Addison-Wesley, 1959).
25. Plass, Gilbert N., "Models for Spectral Band Absorption", J. Opt. Soc. Amer. 48, October, 1958).
26. Griem, H. R., Plasma Spectroscopy, McGraw-Hill Book Co., (1964).

27. Stampa, A., Z. Astrophys. 58-82, (1963).
28. Wray, K. L. and Connolly, T. J., "Nitrogen and Air Radiation in the Near IR", Avco-Everett Research Laboratory Research Report 200 J. Quant. Spect. Rad. Transf. 5, 111-123.
29. Leavitt, B. P., "Temperature and Wavelength Dependence of the Thermal Emission and O - NO Recombination Spectra of NO₂", J. Chem. Phys. 42, No. 43 (Feb. 1965).
30. Branscomb, L. M., Burgh, S. S., Smith, S. J. and Geltman, S., "Photodetachment Cross Section and the Electron Affinity of Atomic Oxygen", Phys. Rev. 111, 504-513 (1958).
31. Taylor, R. L., "Continuum Infrared Radiation from High Temperature Air and Nitrogen", Avco-Everett Research Laboratory Research Report 154, J. Chem. Phys. 39, 2354-2360 (Nov. 1, 1963).
32. Gilmore, F. R., "The Contribution of Generally-Neglected Band Systems and Continua to the Absorption Coefficient of High-Temperature Air", J. Quant. Spect. Rad. Transf. 5, 125-135 (Jan-Feb. 1965).
33. Breeze, J. C., Ferriso, C. C. and Anderson, J. L., "Measurements of the Integrated Intensities of the 5.3- μ Fundamental and 2.7- μ Overtone Bands of NO between 1400° and 2400° K., GDA-DBE64-039 (June 1964).
34. Wurster, W. H., "Measured Transition Probability for the First-Positive Band System of Nitrogen", ARPA Order No. 253-62, CAL Report No. QM-1626-A-3 (January 1962).
35. Wray, K. L. and Teare, J. D., "A Shock Tube Study of the Kinetics of Nitric Oxide at High Temperatures", Avco-Everett Research Laboratory Research Report 95, J. Chem. Phys. 36, 2582-2596, (May, 1961).

<p>Avco-Everett Research Laboratory, Everett, Massachusetts</p> <p>AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION, by R.A. Allen. July 1965. 51 p. incl. illus. (Avco-Everett Research Note 561)</p> <p>(Contracts NAS w-748 and DA-01-021-AMC-12005 (Z))</p> <p>Unclassified report</p> <p>Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000 °K to 23,000 °K and over a density range from 10⁻⁶ to 10 atmospheres. The calculations are presented in a graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic</p> <p>(over)</p>	<p>UNCLASSIFIED</p> <p>I. Air, high temperature — Radiation.</p> <p>II. Title.</p> <p>III. Allen, R. A.</p> <p>IV. Avco-Everett Research Note 561.</p> <p>V. Contract DA-01-021-AMC-12005 (Z).</p> <p>VI. Contract NAS w-748.</p>	<p>Avco-Everett Research Laboratory, Everett, Massachusetts</p> <p>AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION, by R.A. Allen. July 1965. 51 p. incl. illus. (Avco-Everett Research Note 561)</p> <p>(Contracts NAS w-748 and DA-01-021-AMC-12005 (Z))</p> <p>Unclassified report</p> <p>Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000 °K to 23,000 °K and over a density range from 10⁻⁶ to 10 atmospheres. The calculations are presented in a graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic</p> <p>(over)</p>	<p>UNCLASSIFIED</p> <p>I. Air, high temperature — Radiation.</p> <p>II. Title.</p> <p>III. Allen, R. A.</p> <p>IV. Avco-Everett Research Note 561.</p> <p>V. Contract DA-01-021-AMC-12005 (Z).</p> <p>VI. Contract NAS w-748.</p>	<p>UNCLASSIFIED</p> <p>line and free-bound transitions are important contributors at elevated temperatures and also generally undergo significant amounts of self-absorption. Consequently calculations for these two classes of radiation have been made with slab thickness as an added parameter. The lines, free-bound and free-free radiation have also been split into the two spectral regions for the radiation flux above and below 2000 Å.</p>	<p>UNCLASSIFIED</p>
<p>Avco-Everett Research Laboratory, Everett, Massachusetts</p> <p>AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION, by R.A. Allen. July 1965. 51 p. incl. illus. (Avco-Everett Research Note 561)</p> <p>(Contracts NAS w-748 and DA-01-021-AMC-12005 (Z))</p> <p>Unclassified report</p> <p>Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000 °K to 23,000 °K and over a density range from 10⁻⁶ to 10 atmospheres. The calculations are presented in a graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic</p> <p>(over)</p>	<p>UNCLASSIFIED</p> <p>I. Air, high temperature — Radiation.</p> <p>II. Title.</p> <p>III. Allen, R. A.</p> <p>IV. Avco-Everett Research Note 561.</p> <p>V. Contract DA-01-021-AMC-12005 (Z).</p> <p>VI. Contract NAS w-748.</p>	<p>Avco-Everett Research Laboratory, Everett, Massachusetts</p> <p>AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION, by R.A. Allen. July 1965. 51 p. incl. illus. (Avco-Everett Research Note 561)</p> <p>(Contracts NAS w-748 and DA-01-021-AMC-12005 (Z))</p> <p>Unclassified report</p> <p>Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000 °K to 23,000 °K and over a density range from 10⁻⁶ to 10 atmospheres. The calculations are presented in a graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic</p> <p>(over)</p>	<p>UNCLASSIFIED</p> <p>I. Air, high temperature — Radiation.</p> <p>II. Title.</p> <p>III. Allen, R. A.</p> <p>IV. Avco-Everett Research Note 561.</p> <p>V. Contract DA-01-021-AMC-12005 (Z).</p> <p>VI. Contract NAS w-748.</p>	<p>UNCLASSIFIED</p> <p>line and free-bound transitions are important contributors at elevated temperatures and also generally undergo significant amounts of self-absorption. Consequently calculations for these two classes of radiation have been made with slab thickness as an added parameter. The lines, free-bound and free-free radiation have also been split into the two spectral regions for the radiation flux above and below 2000 Å.</p>	<p>UNCLASSIFIED</p>

<p>Avco-Everett Research Laboratory, Everett, Massachusetts</p> <p>AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION, by R.A. Allen. July 1965. 51 p. incl. illus. (Avco-Everett Research Note 561)</p> <p>(Contracts NAS w-748 and DA-01-021-AMC-12005(Z))</p> <p>Unclassified report</p> <p>Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000°K to 23,000°K and over a density range from 10⁻⁶ to 10 atmospheres. The calculations are presented in a graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic</p> <p>(over)</p>	<p>UNCLASSIFIED</p> <p>I. Air, high temperature — Radiation. Title.</p> <p>II. Allen, R. A.</p> <p>III. Avco-Everett Research Note 561.</p> <p>IV. Contract DA-01-021-AMC-12005(Z).</p> <p>V. Contract NAS w-748.</p>
<p>Avco-Everett Research Laboratory, Everett, Massachusetts</p> <p>AIR RADIATION GRAPHS: SPECTRALLY INTEGRATED FLUXES INCLUDING LINE CONTRIBUTIONS AND SELF ABSORPTION, by R.A. Allen. July 1965. 51 p. incl. illus. (Avco-Everett Research Note 561)</p> <p>(Contracts NAS w-748 and DA-01-021-AMC-12005(Z))</p> <p>Unclassified report</p> <p>Calculations have been made of the equilibrium radiation emitted from air over a range of temperatures from 2,000°K to 23,000°K and over a density range from 10⁻⁶ to 10 atmospheres. The calculations are presented in a graphical form and are in terms of the radiation flux emitted from one side of infinite slabs of finite thickness. The molecular band systems have been calculated using a smeared rotational model, and are carried out for a slab thickness of 1 cm. Atomic</p> <p>(over)</p>	<p>UNCLASSIFIED</p> <p>I. Air, high temperature — Radiation. Title.</p> <p>II. Allen, R. A.</p> <p>III. Avco-Everett Research Note 561.</p> <p>IV. Contract DA-01-021-AMC-12005(Z).</p> <p>V. Contract NAS w-748.</p>
<p>line and free-bound transitions are important contributors at elevated temperatures and also generally undergo significant amounts of self-absorption. Consequently calculations for these two classes of radiation have been made with slab thickness as an added parameter. The lines, free-bound and free-free radiation have also been split into the two spectral regions for the radiation flux above and below 2000 Å.</p>	<p>UNCLASSIFIED</p> <p>line and free-bound transitions are important contributors at elevated temperatures and also generally undergo significant amounts of self-absorption. Consequently calculations for these two classes of radiation have been made with slab thickness as an added parameter. The lines, free-bound and free-free radiation have also been split into the two spectral regions for the radiation flux above and below 2000 Å.</p>